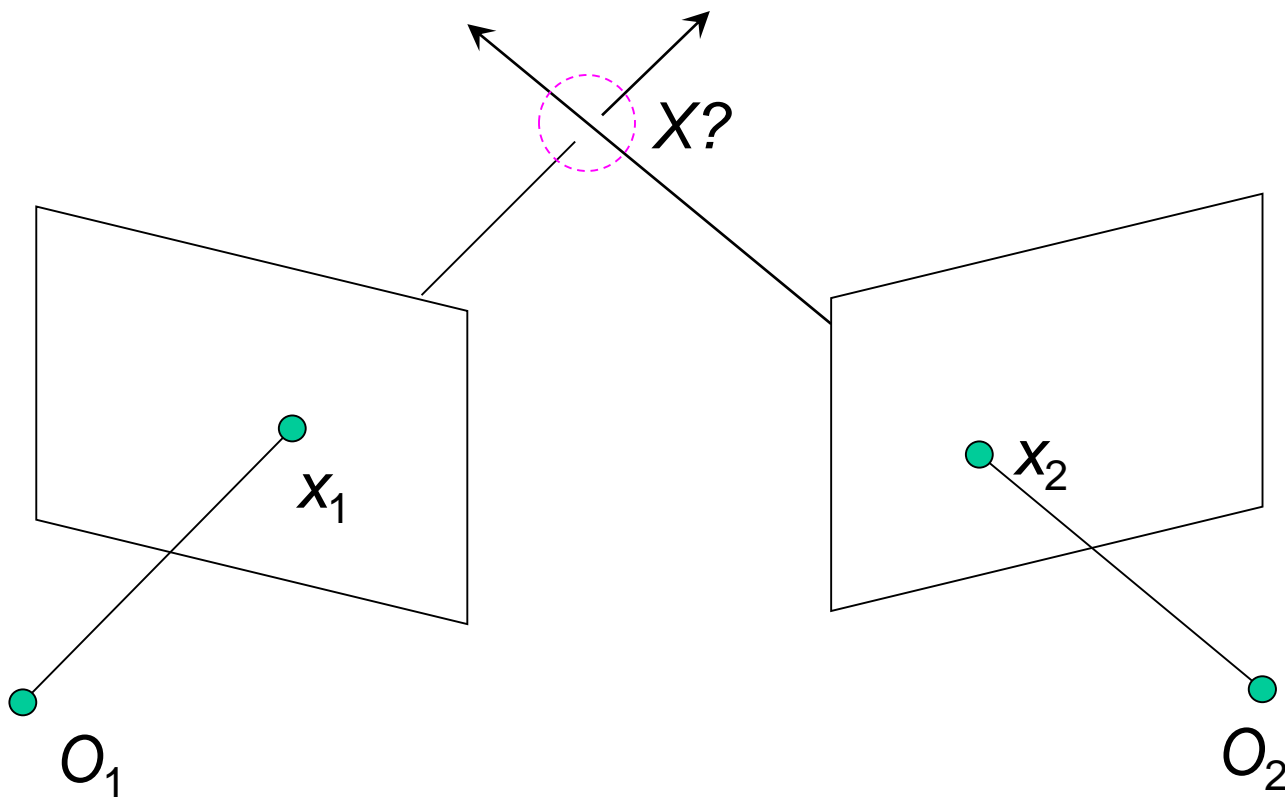


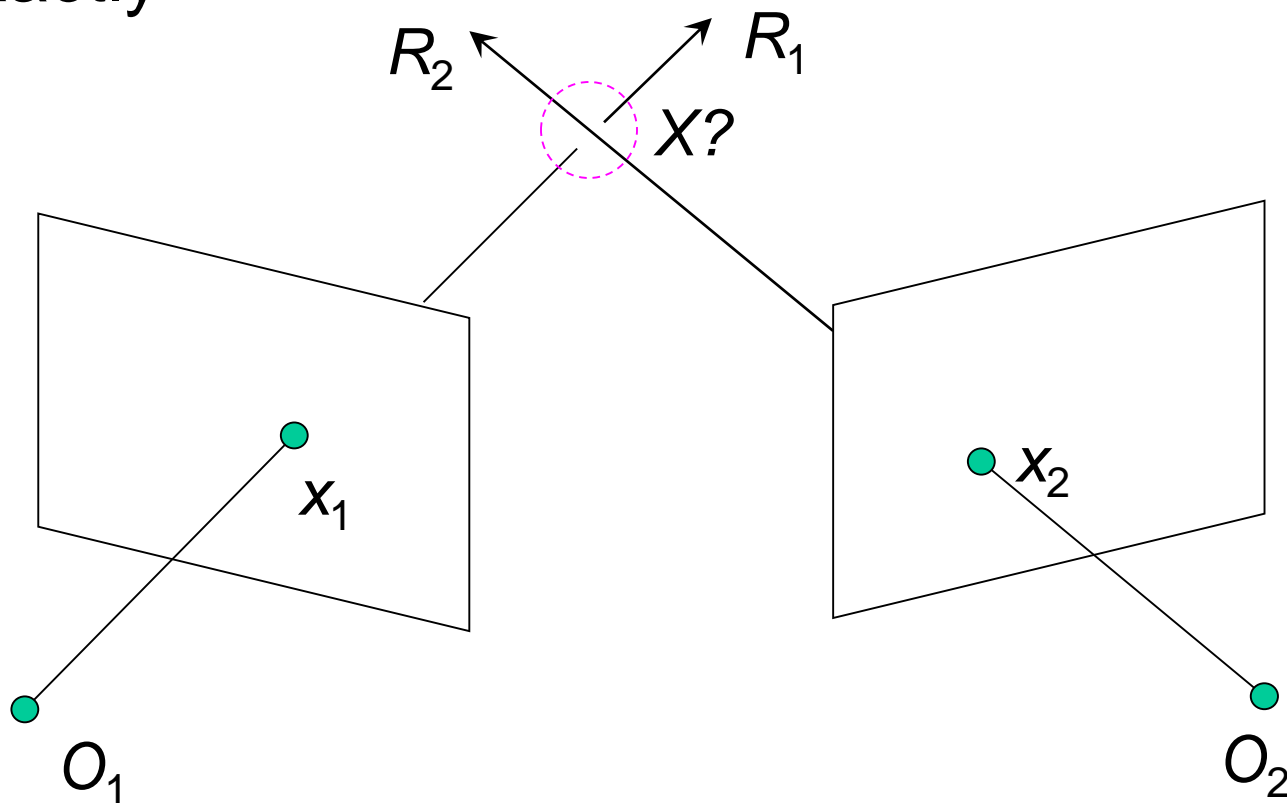
Triangulation

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



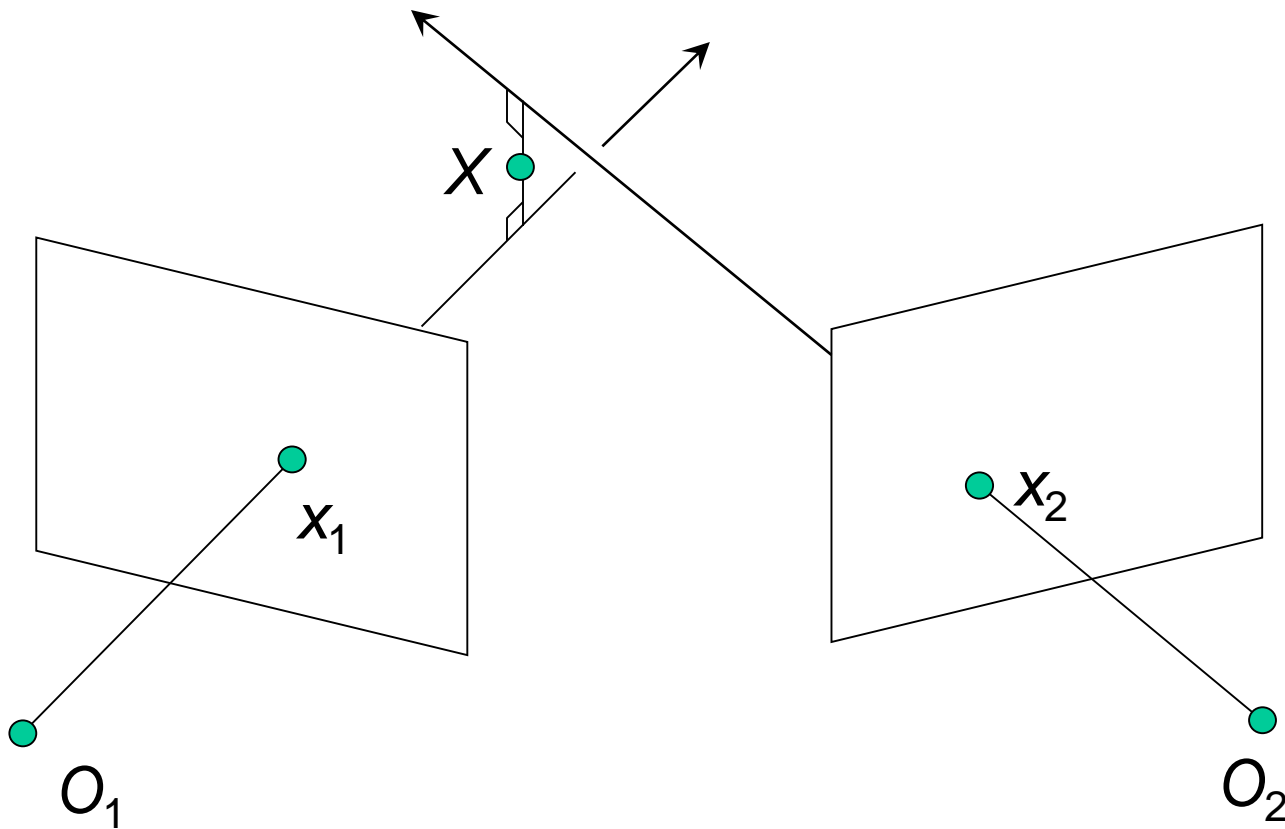
Triangulation

- We want to intersect the two visual rays corresponding to x_1 and x_2 , but because of noise and numerical errors, they don't meet exactly



Triangulation: Geometric approach

- Find shortest segment connecting the two viewing rays and let X be the midpoint of that segment



Triangulation: Linear approach

$$\lambda_1 \mathbf{x}_1 = \mathbf{P}_1 \mathbf{X}$$

$$\lambda_2 \mathbf{x}_2 = \mathbf{P}_2 \mathbf{X}$$

Triangulation: Linear approach

$$\lambda_1 \mathbf{x}_1 = \mathbf{P}_1 \mathbf{X} \quad \mathbf{x}_1 \times \mathbf{P}_1 \mathbf{X} = 0$$

$$\lambda_2 \mathbf{x}_2 = \mathbf{P}_2 \mathbf{X} \quad \mathbf{x}_2 \times \mathbf{P}_2 \mathbf{X} = 0$$

Triangulation: Linear approach

$$\lambda_1 \mathbf{x}_1 = \mathbf{P}_1 \mathbf{X} \quad \mathbf{x}_1 \times \mathbf{P}_1 \mathbf{X} = 0$$

$$\lambda_2 \mathbf{x}_2 = \mathbf{P}_2 \mathbf{X} \quad \mathbf{x}_2 \times \mathbf{P}_2 \mathbf{X} = 0$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times] \mathbf{b}$$

Triangulation: Linear approach

$$\begin{array}{lll} \lambda_1 \mathbf{x}_1 = \mathbf{P}_1 \mathbf{X} & \mathbf{x}_1 \times \mathbf{P}_1 \mathbf{X} = 0 & [\mathbf{x}_{1 \times}] \mathbf{P}_1 \mathbf{X} = 0 \\ \lambda_2 \mathbf{x}_2 = \mathbf{P}_2 \mathbf{X} & \mathbf{x}_2 \times \mathbf{P}_2 \mathbf{X} = 0 & [\mathbf{x}_{2 \times}] \mathbf{P}_2 \mathbf{X} = 0 \end{array}$$

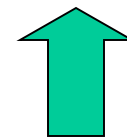
Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_{\times}] \mathbf{b}$$

Triangulation: Linear approach

$$\lambda_1 \mathbf{x}_1 = \mathbf{P}_1 \mathbf{X} \quad \mathbf{x}_1 \times \mathbf{P}_1 \mathbf{X} = 0 \quad [\mathbf{x}_{1 \times}] \mathbf{P}_1 \mathbf{X} = 0$$

$$\lambda_2 \mathbf{x}_2 = \mathbf{P}_2 \mathbf{X} \quad \mathbf{x}_2 \times \mathbf{P}_2 \mathbf{X} = 0 \quad [\mathbf{x}_{2 \times}] \mathbf{P}_2 \mathbf{X} = 0$$

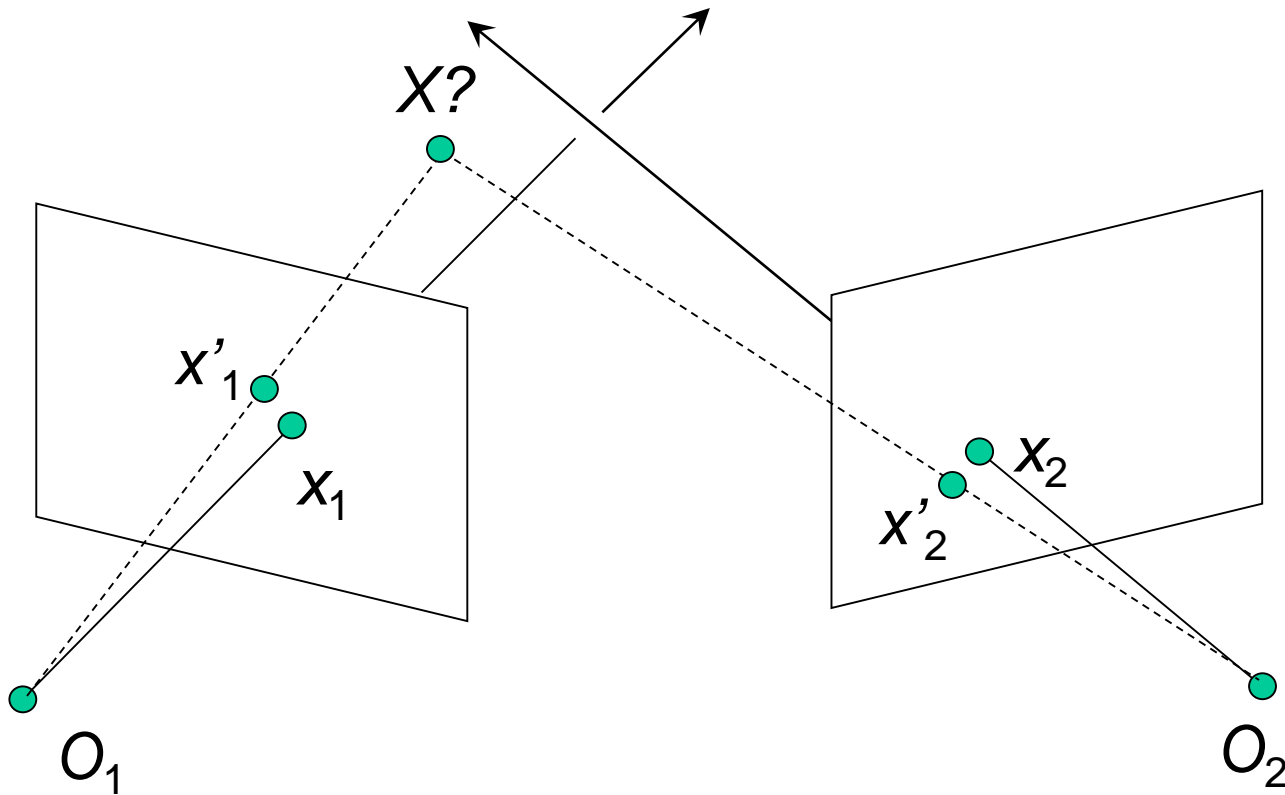


Two independent equations each in terms of three unknown entries of \mathbf{X}

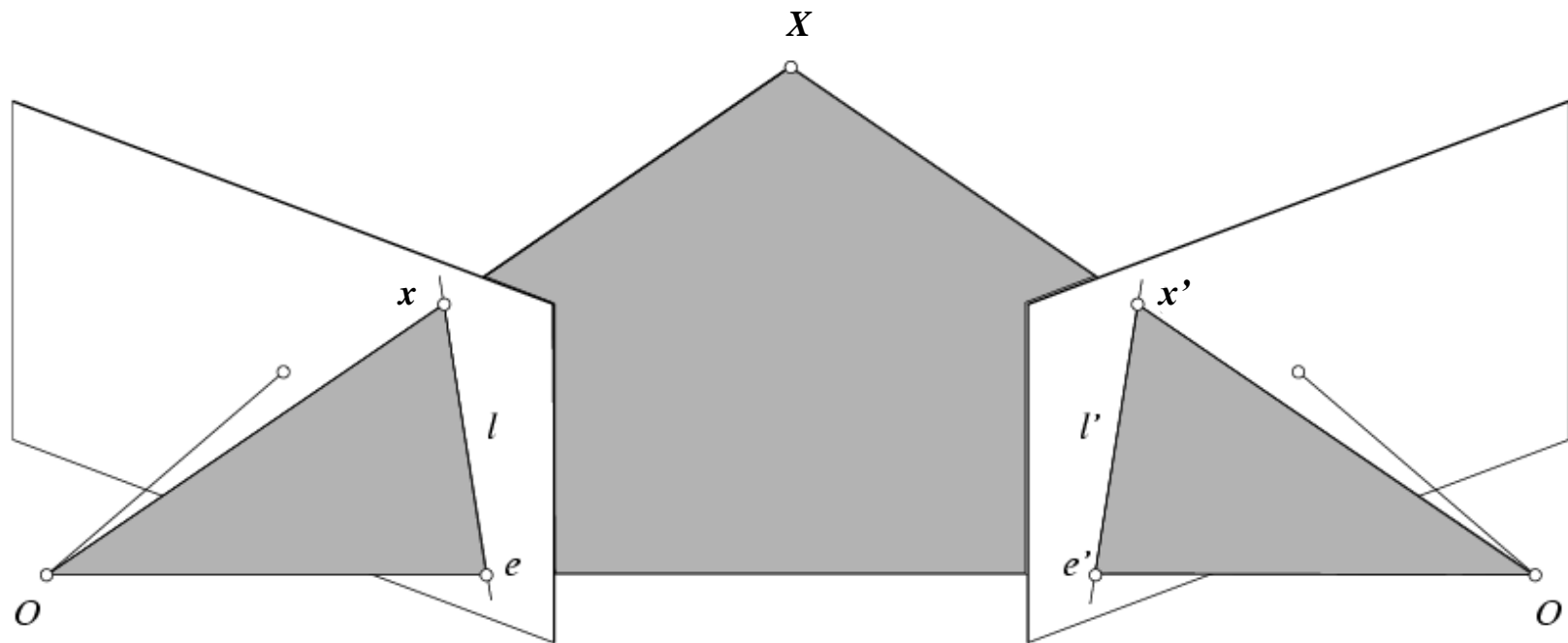
Triangulation: Nonlinear approach

Find X that minimizes

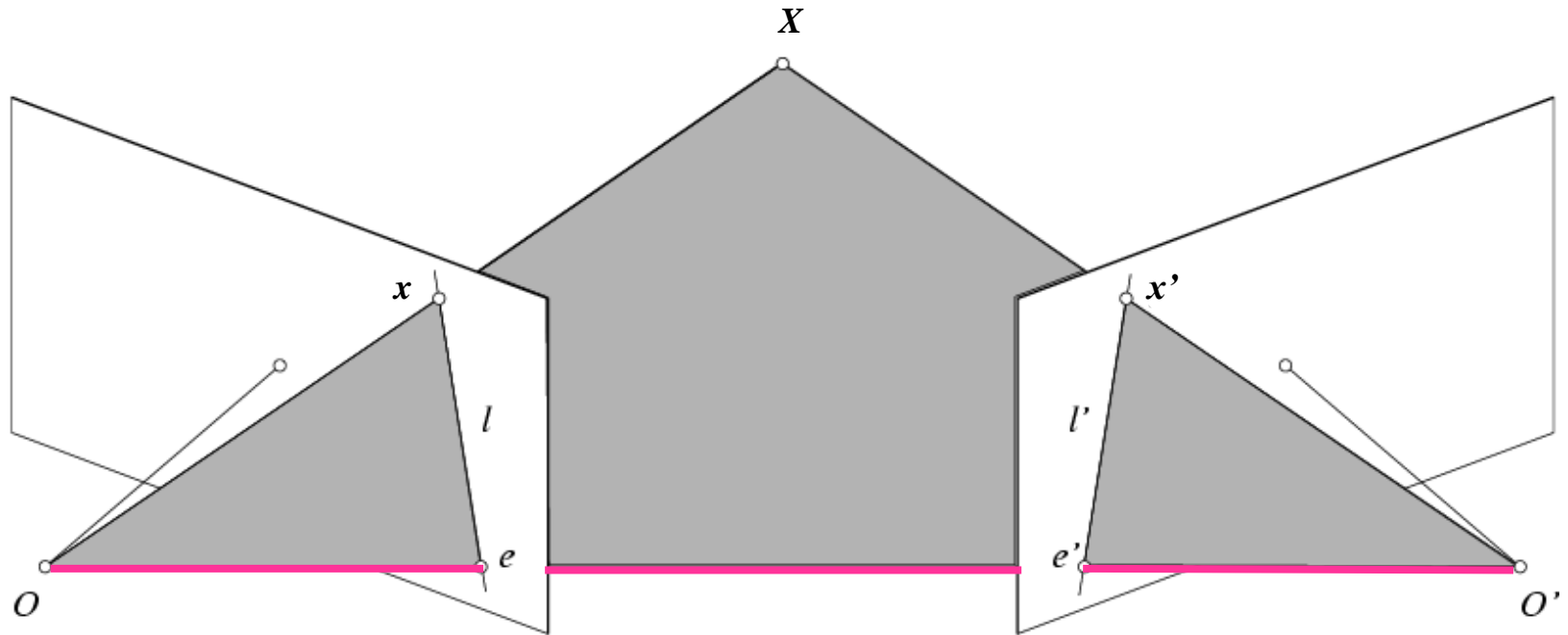
$$d^2(x_1, P_1 X) + d^2(x_2, P_2 X)$$



Epipolar geometry

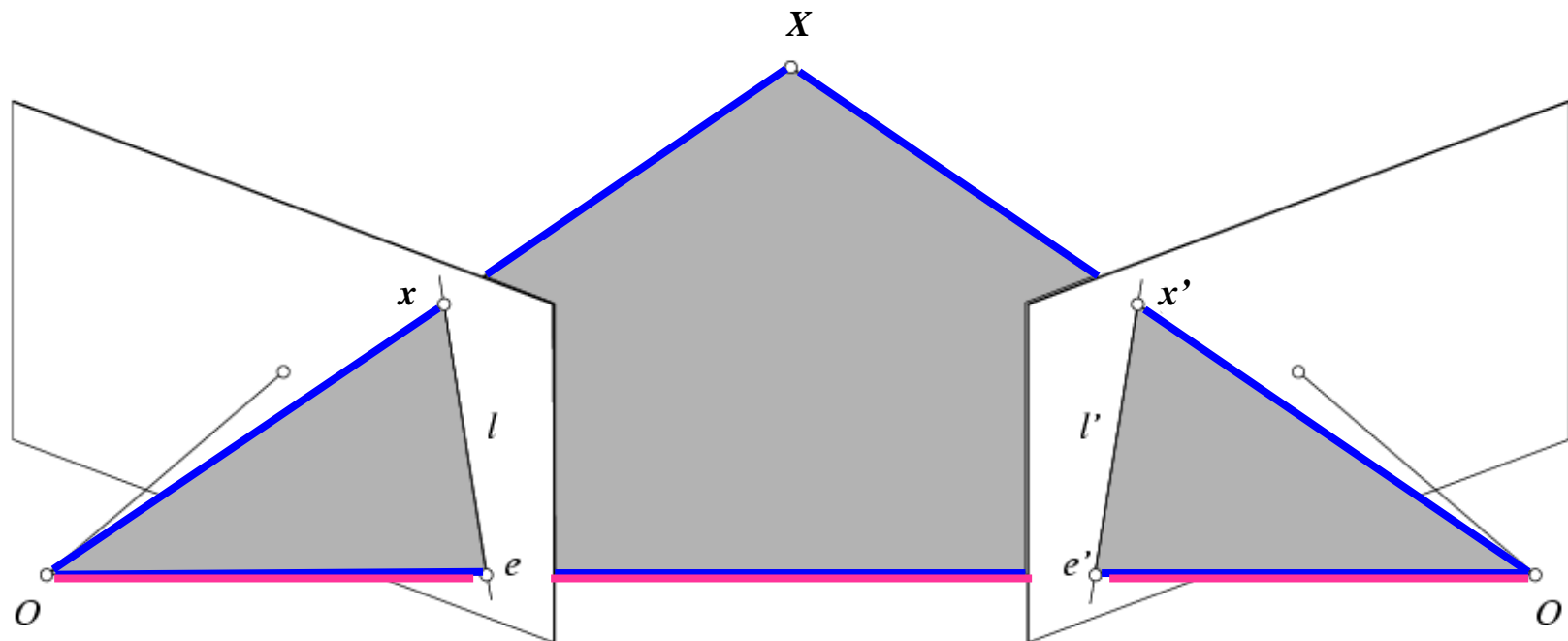


Epipolar geometry



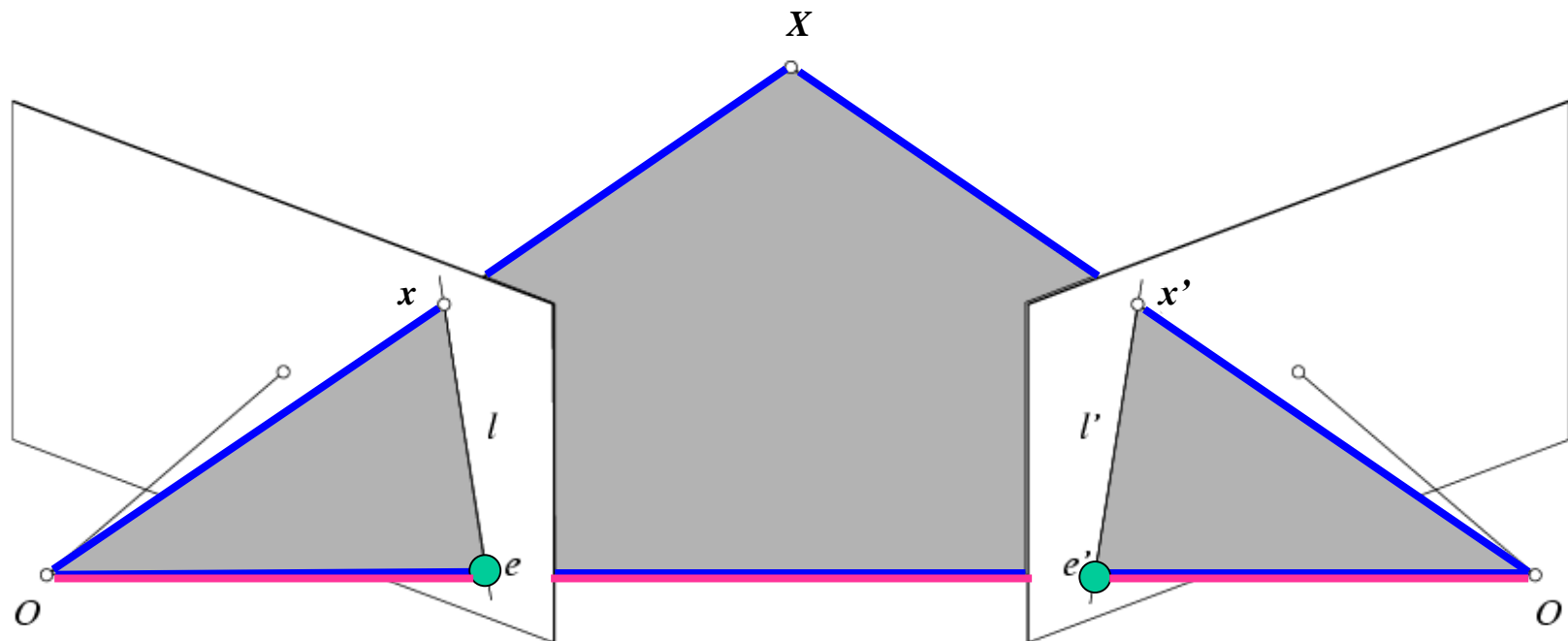
- **Baseline** – line connecting the two camera centers

Epipolar geometry



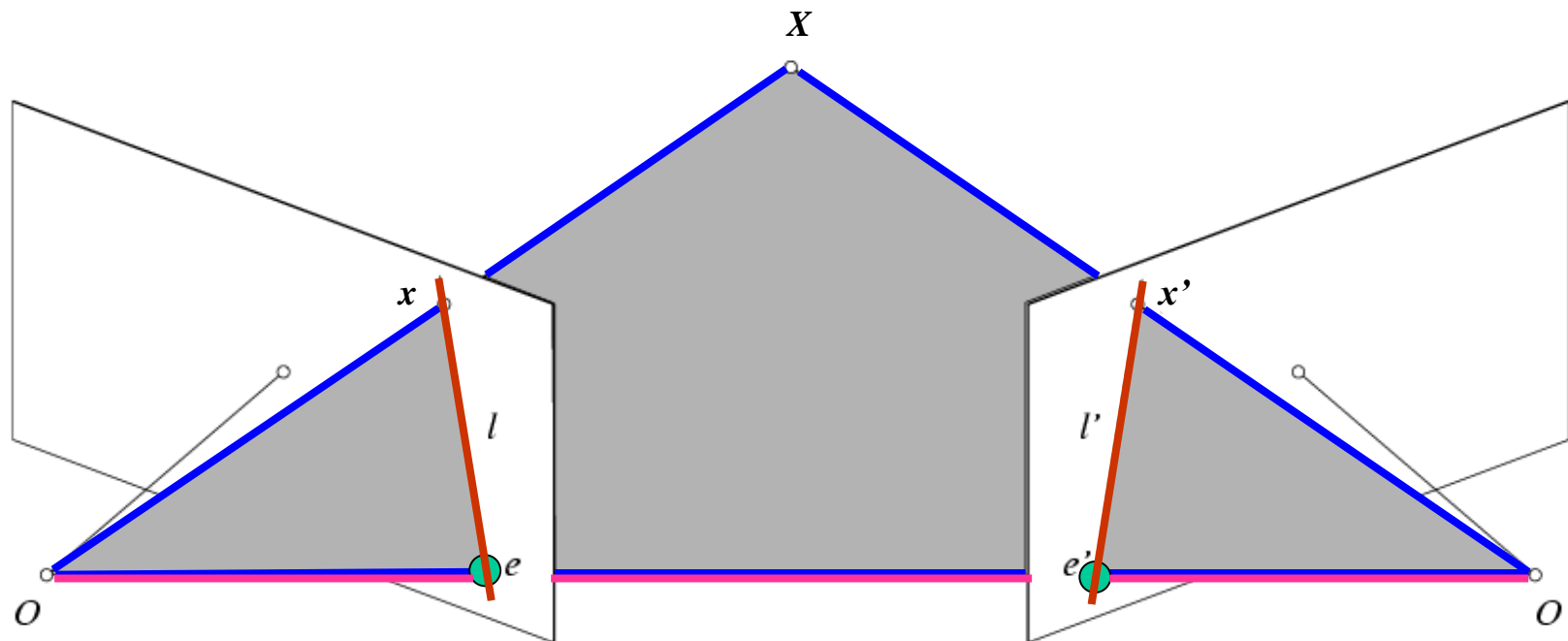
- **Baseline** – line connecting the two camera centers
- **Epipolar Plane** – plane containing baseline (1D family)

Epipolar geometry



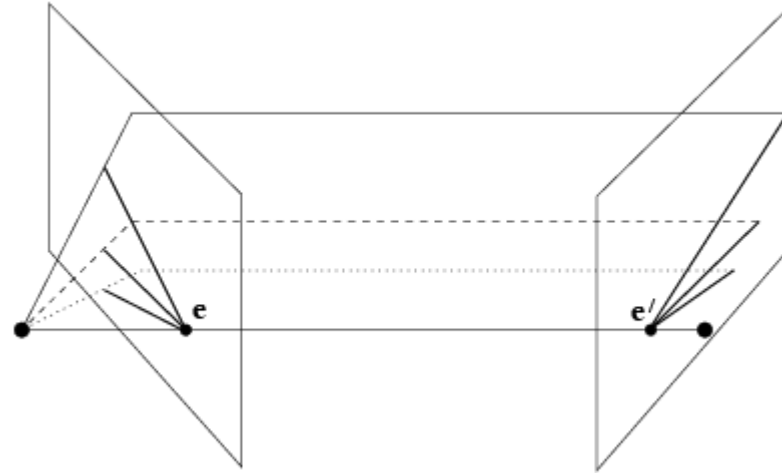
- **Baseline** – line connecting the two camera centers
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipoles**
 - = intersections of baseline with image planes
 - = projections of the other camera center
 - = vanishing points of camera motion direction

Epipolar geometry

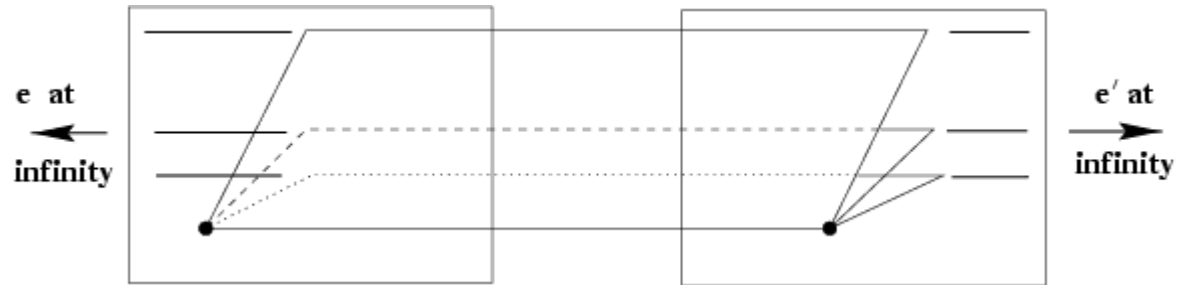


- **Baseline** – line connecting the two camera centers
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipoles**
 - = intersections of baseline with image planes
 - = projections of the other camera center
 - = vanishing points of camera motion direction
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

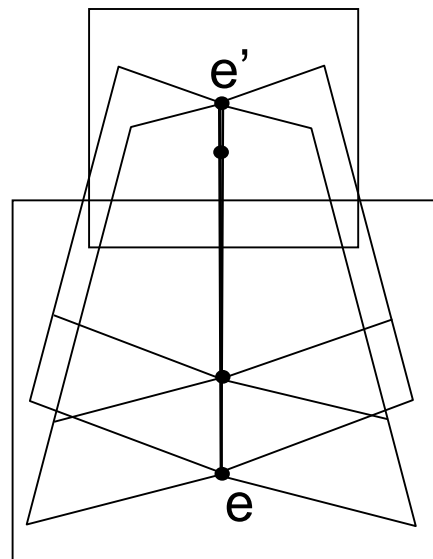
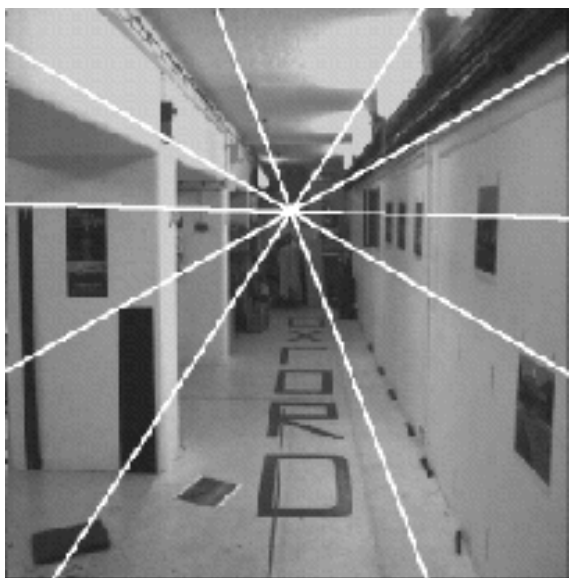
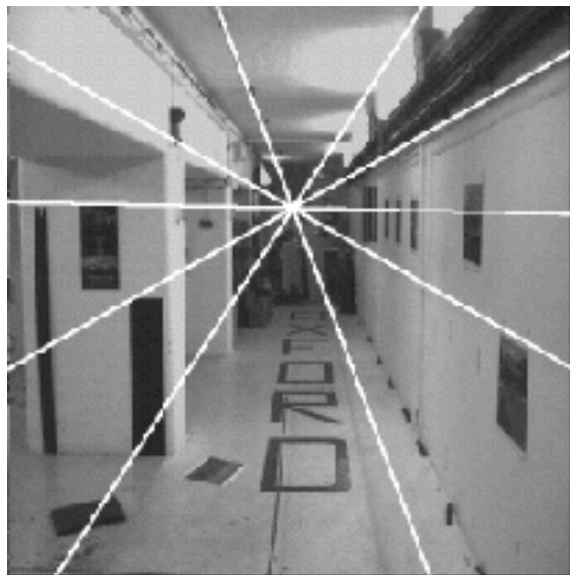
Example: Converging cameras



Example: Motion parallel to image plane



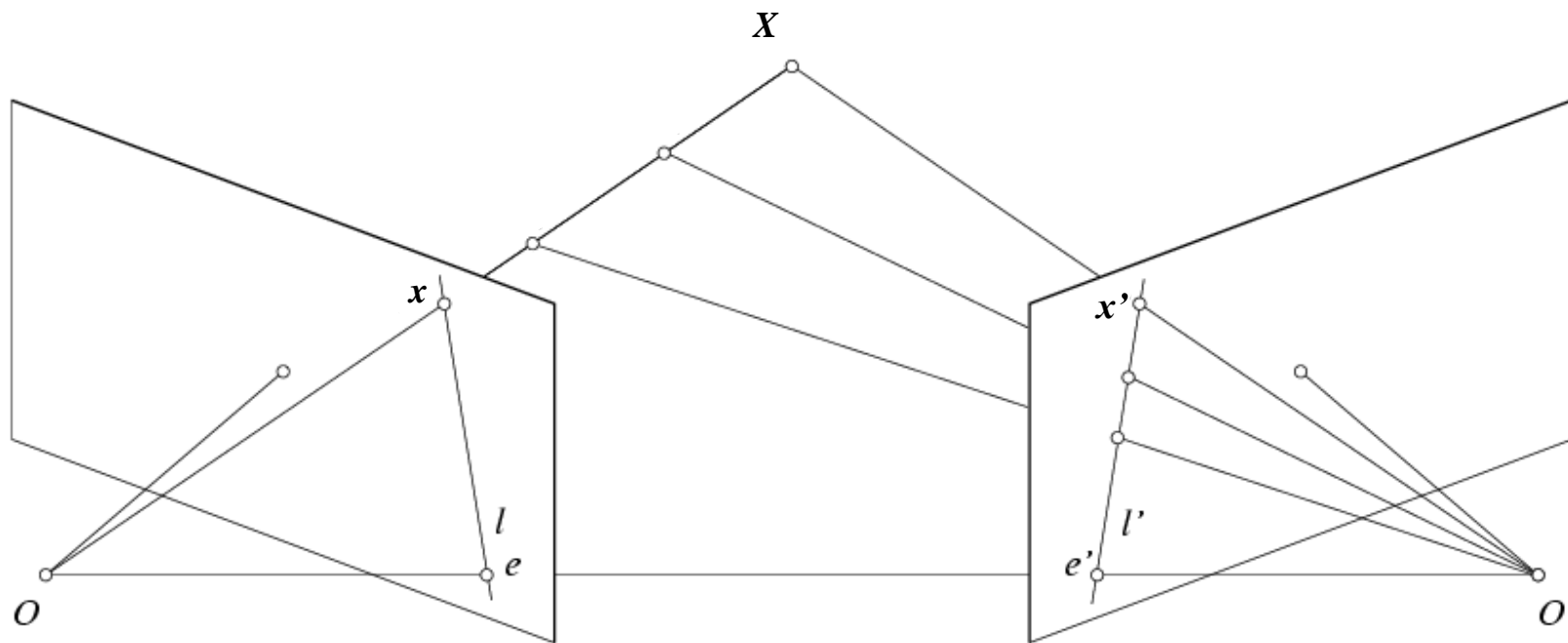
Example: Forward motion



Epipole has same coordinates in both images.

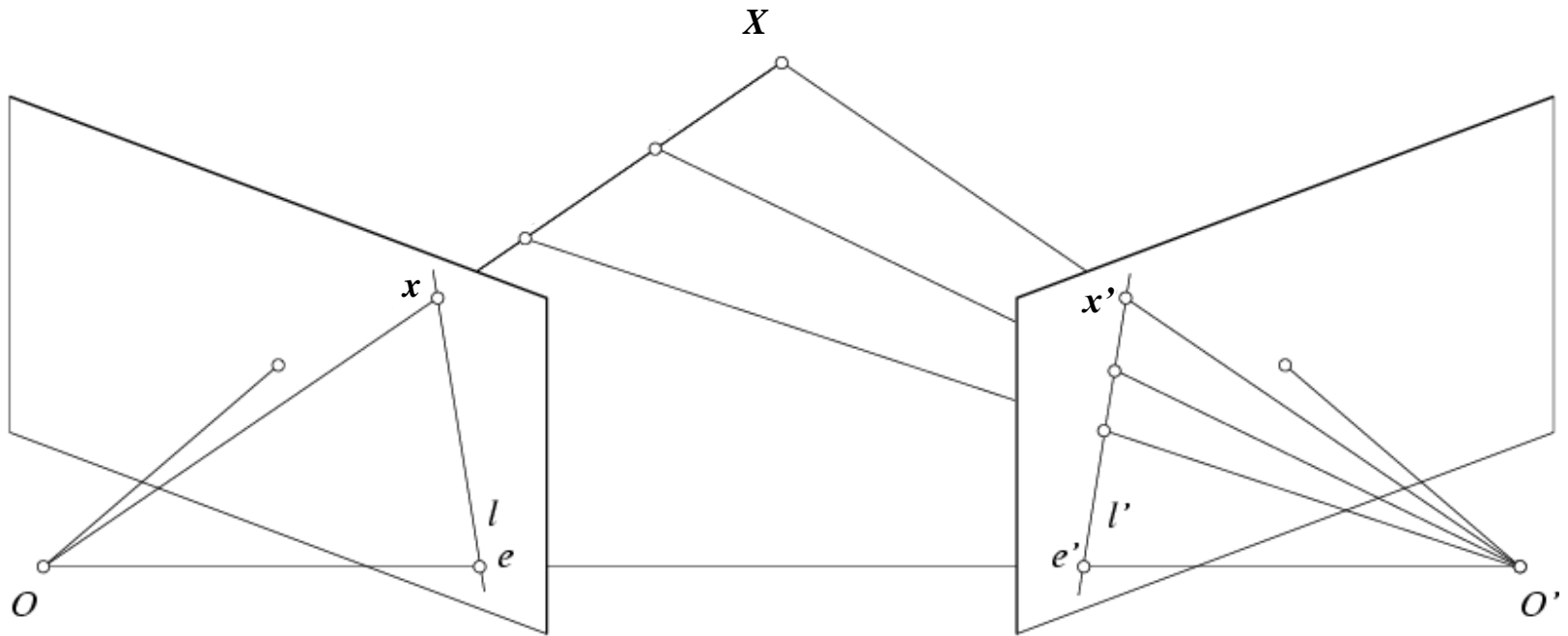
Points move along lines radiating from e : “Focus of expansion”

Epipolar constraint

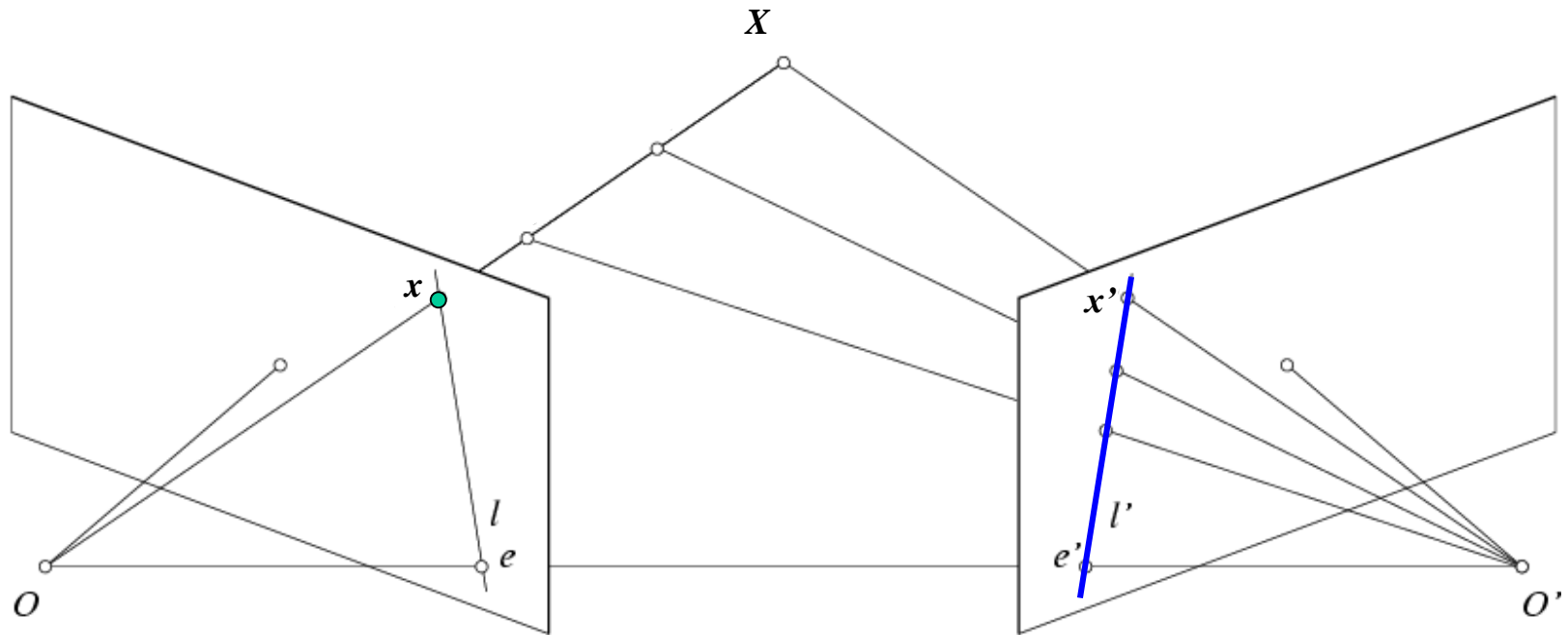


- If we observe a point x in one image, where can the corresponding point x' be in the other image?

Epipolar constraint

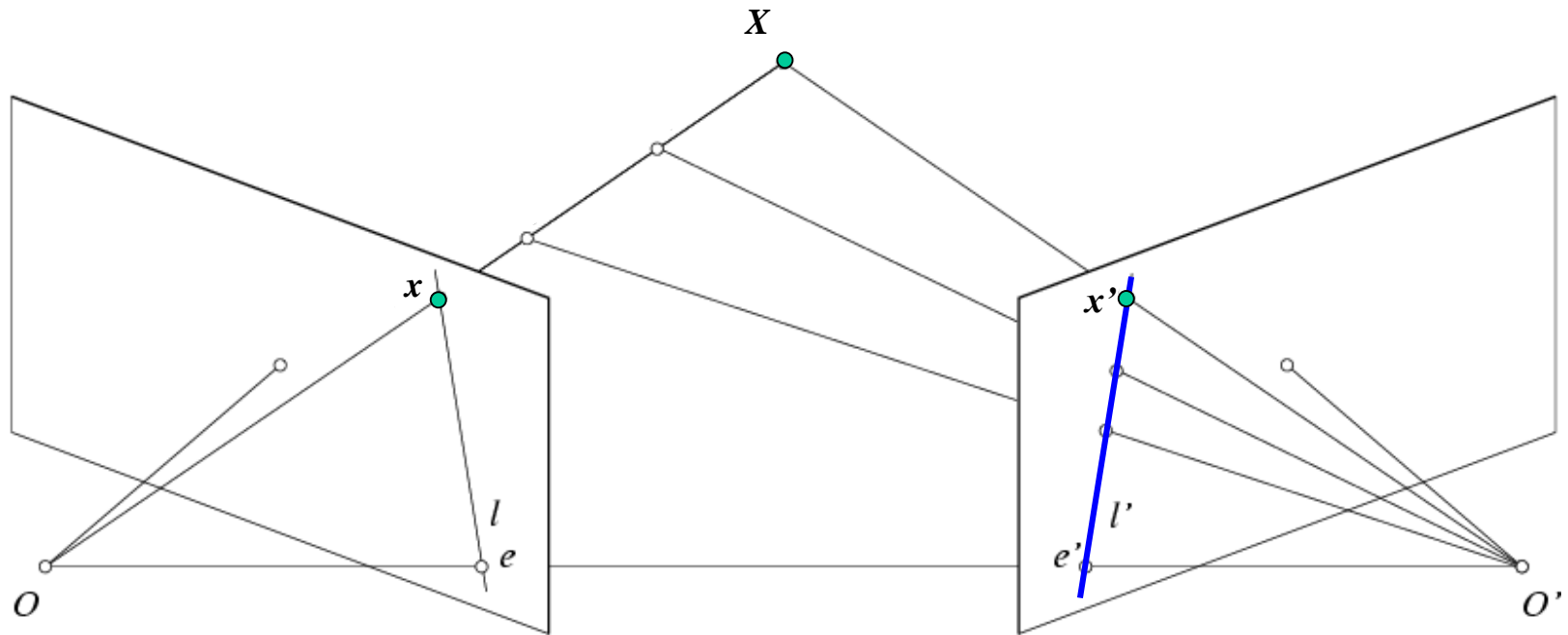


Epipolar constraint



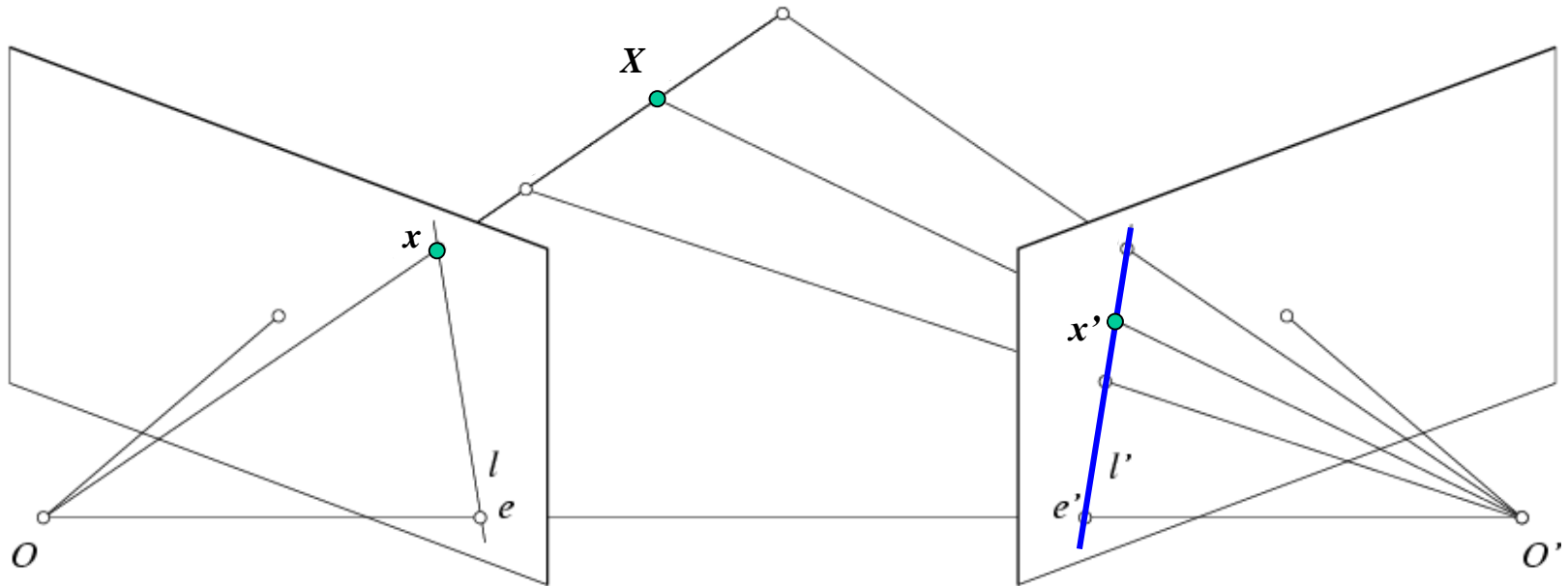
- Potential matches for x have to lie on the corresponding epipolar line l' .

Epipolar constraint



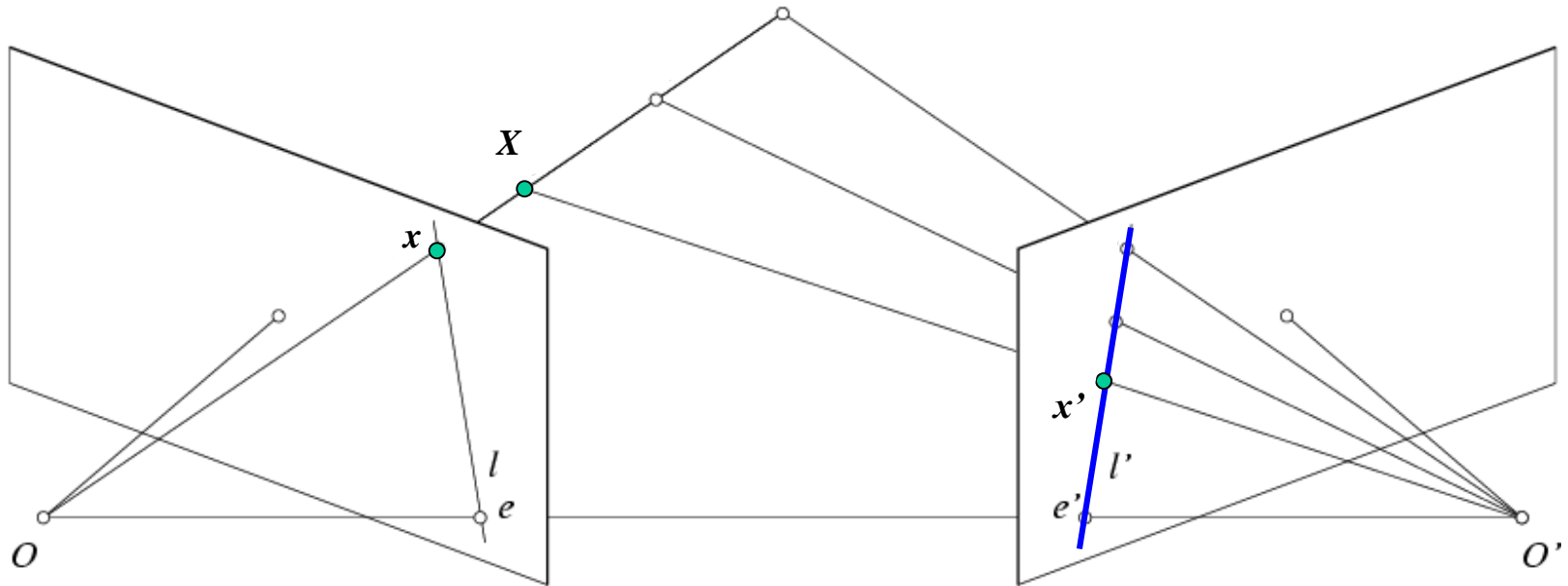
- Potential matches for x have to lie on the corresponding epipolar line l' .

Epipolar constraint



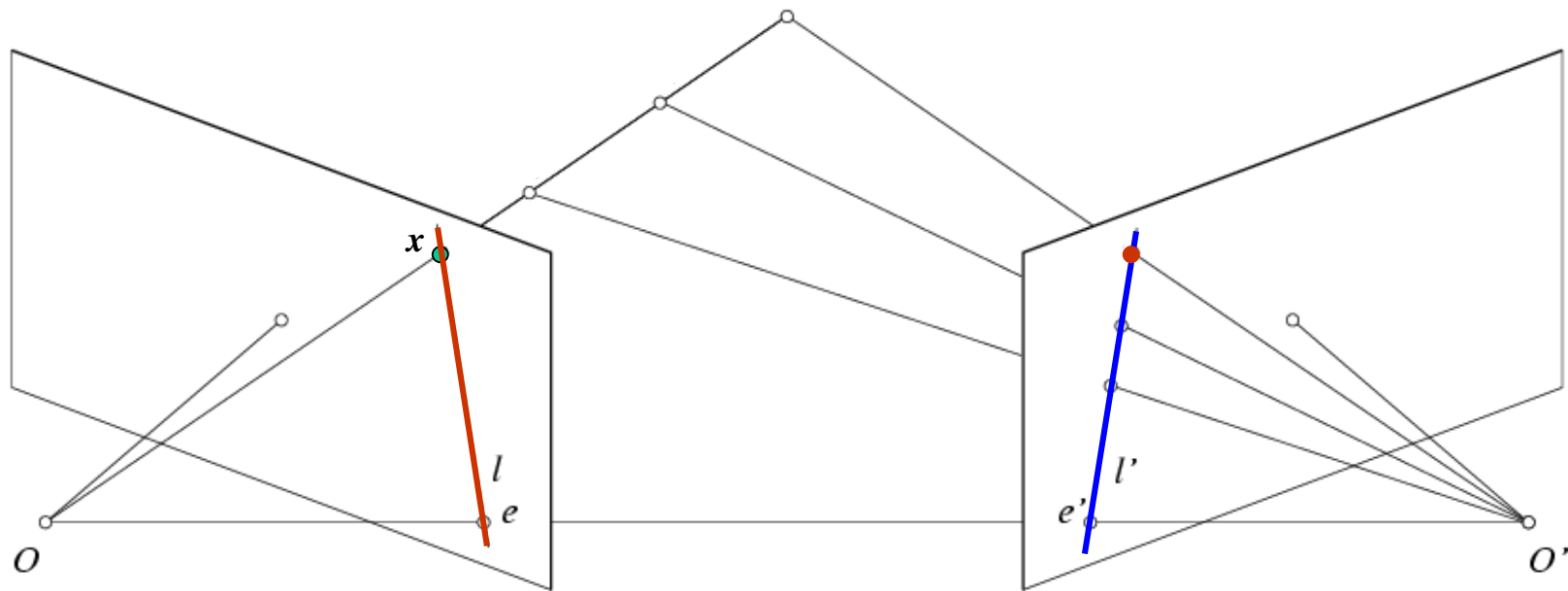
- Potential matches for x have to lie on the corresponding epipolar line l' .

Epipolar constraint



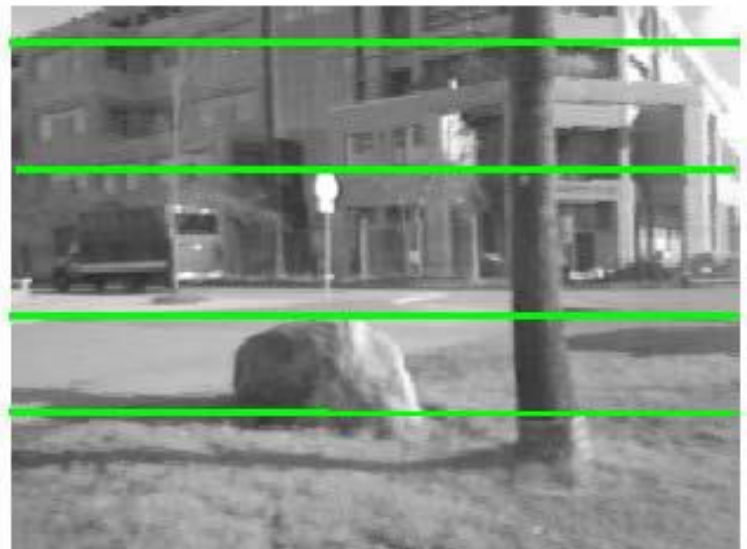
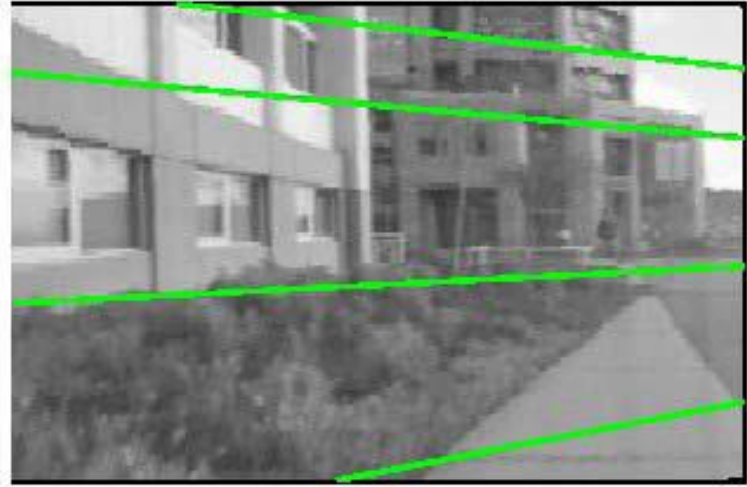
- Potential matches for x have to lie on the corresponding epipolar line l' .

Epipolar constraint

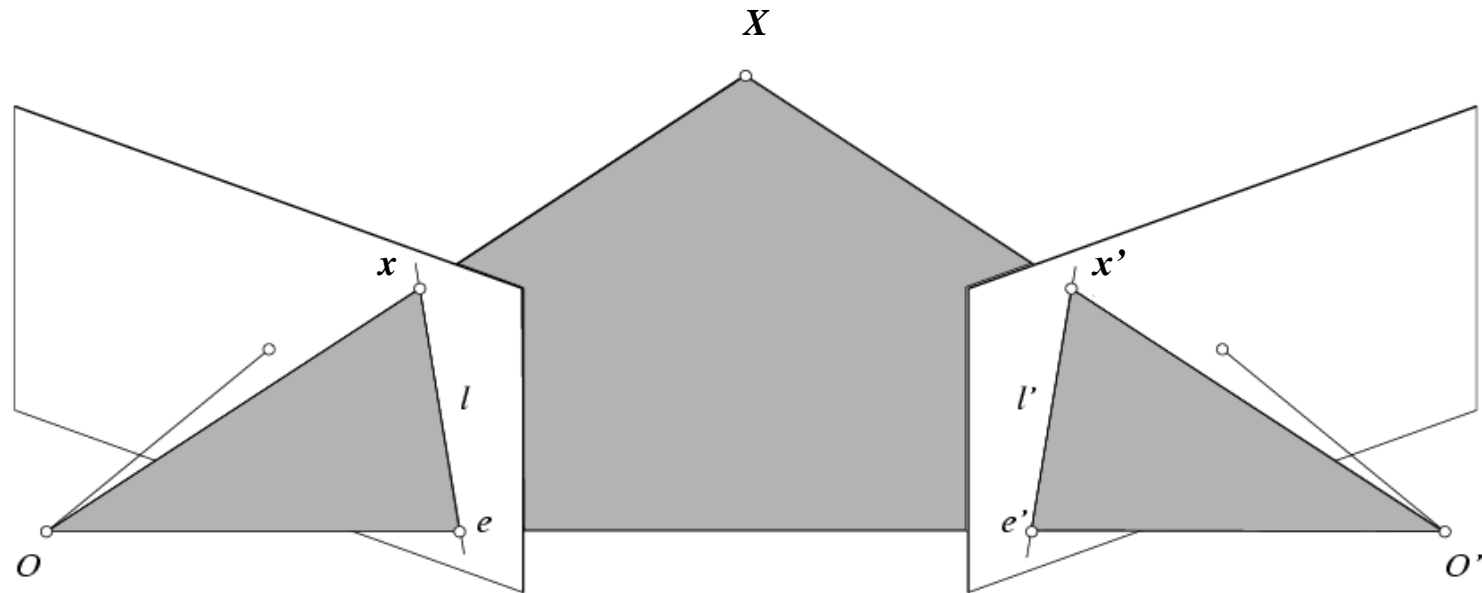


- Potential matches for x have to lie on the corresponding epipolar line l' .
- Potential matches for x' have to lie on the corresponding epipolar line l .

Epipolar constraint example

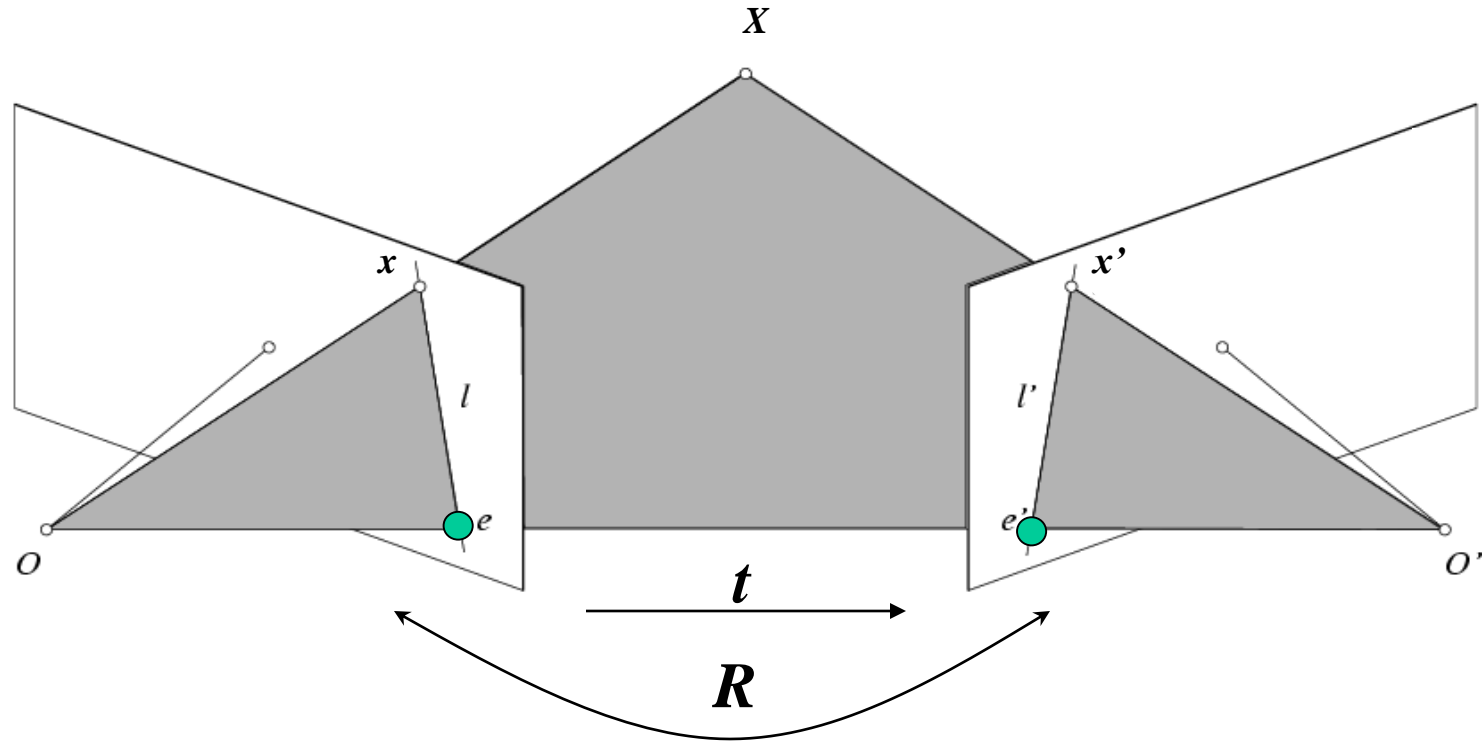


Epipolar constraint: Calibrated case

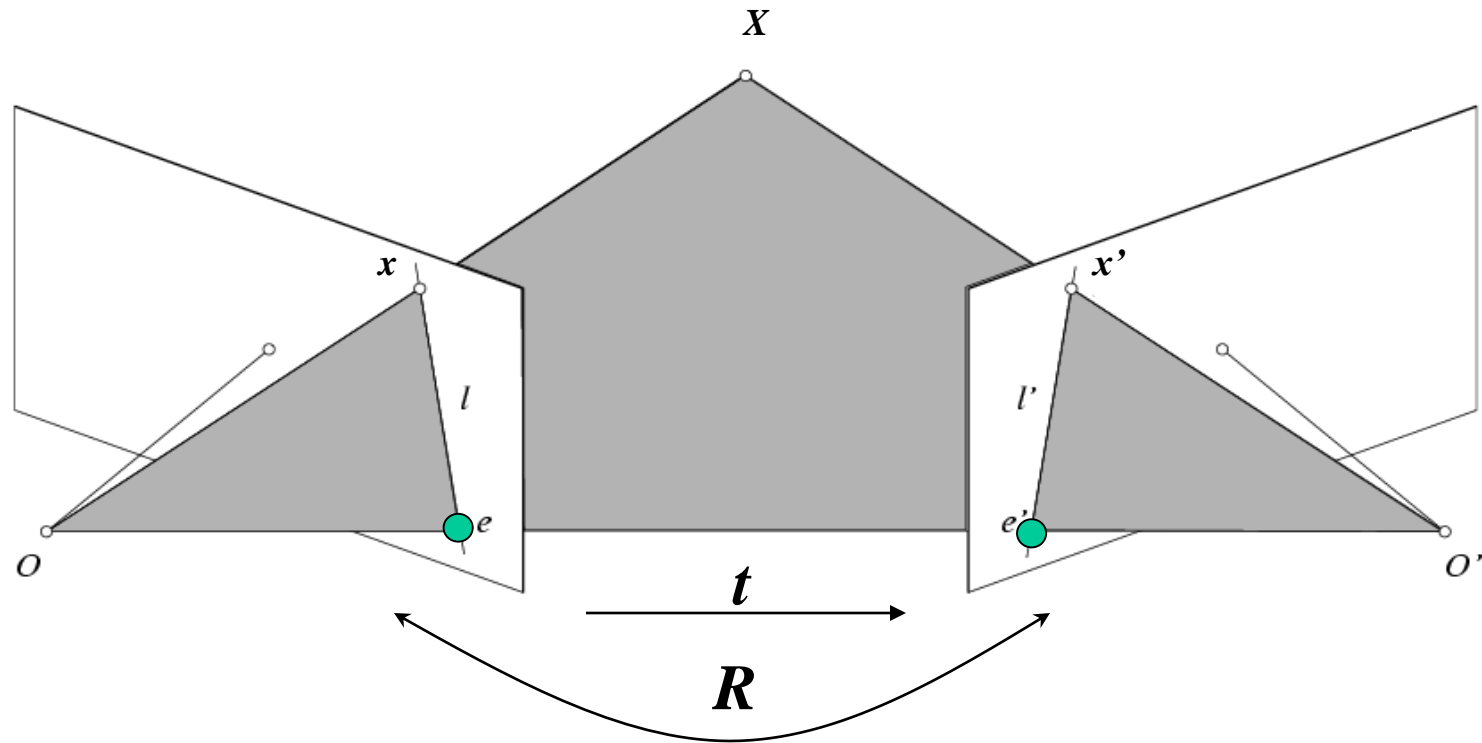


- Assume that the intrinsic and extrinsic parameters of the cameras are known
- We can multiply the projection matrix of each camera (and the image points) by the inverse of the calibration matrix to get *normalized* image coordinates
- We can also set the global coordinate system to the coordinate system of the first camera

Epipolar constraint: Calibrated case



Epipolar constraint: Calibrated case

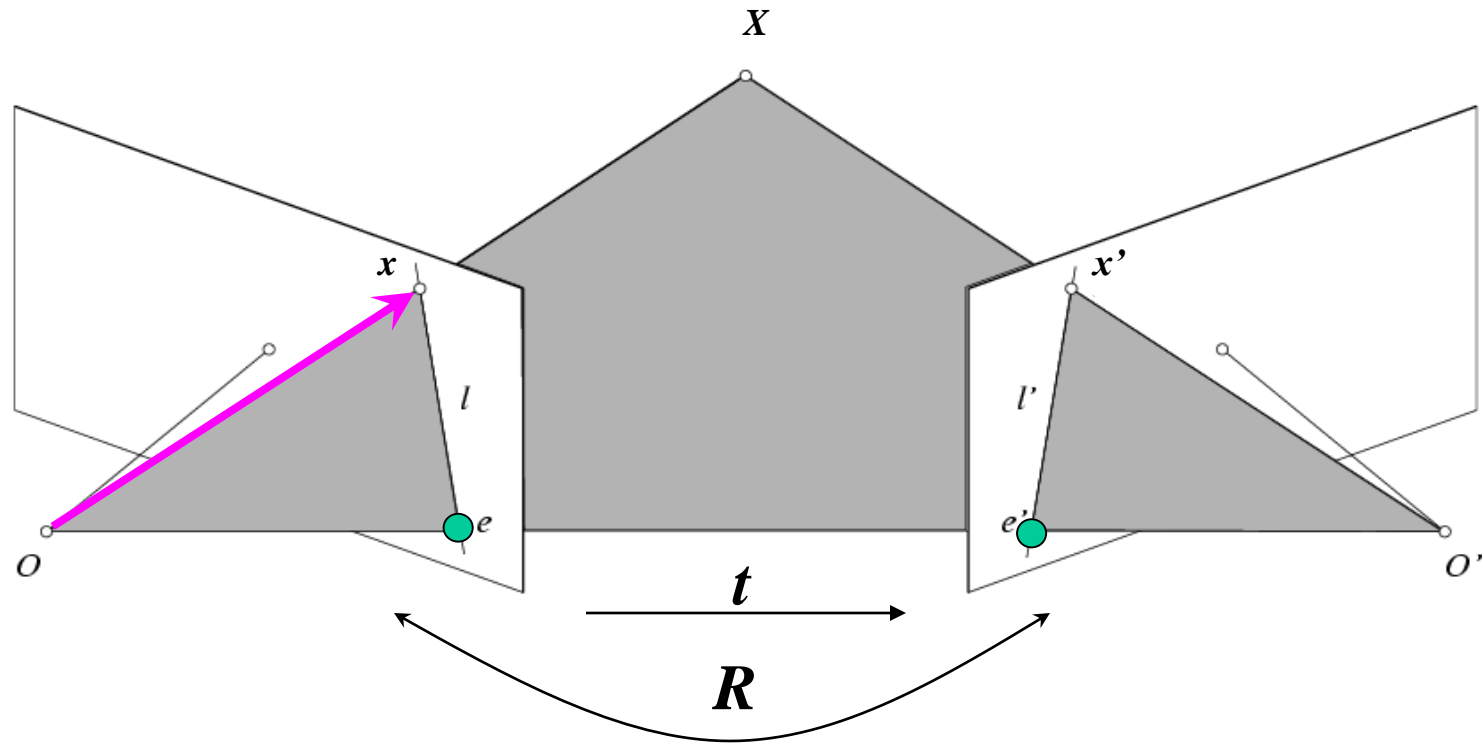


Camera matrix: $[I|0]$

$$X = (u, v, w, 1)^T$$

$$x = (u, v, w)^T$$

Epipolar constraint: Calibrated case

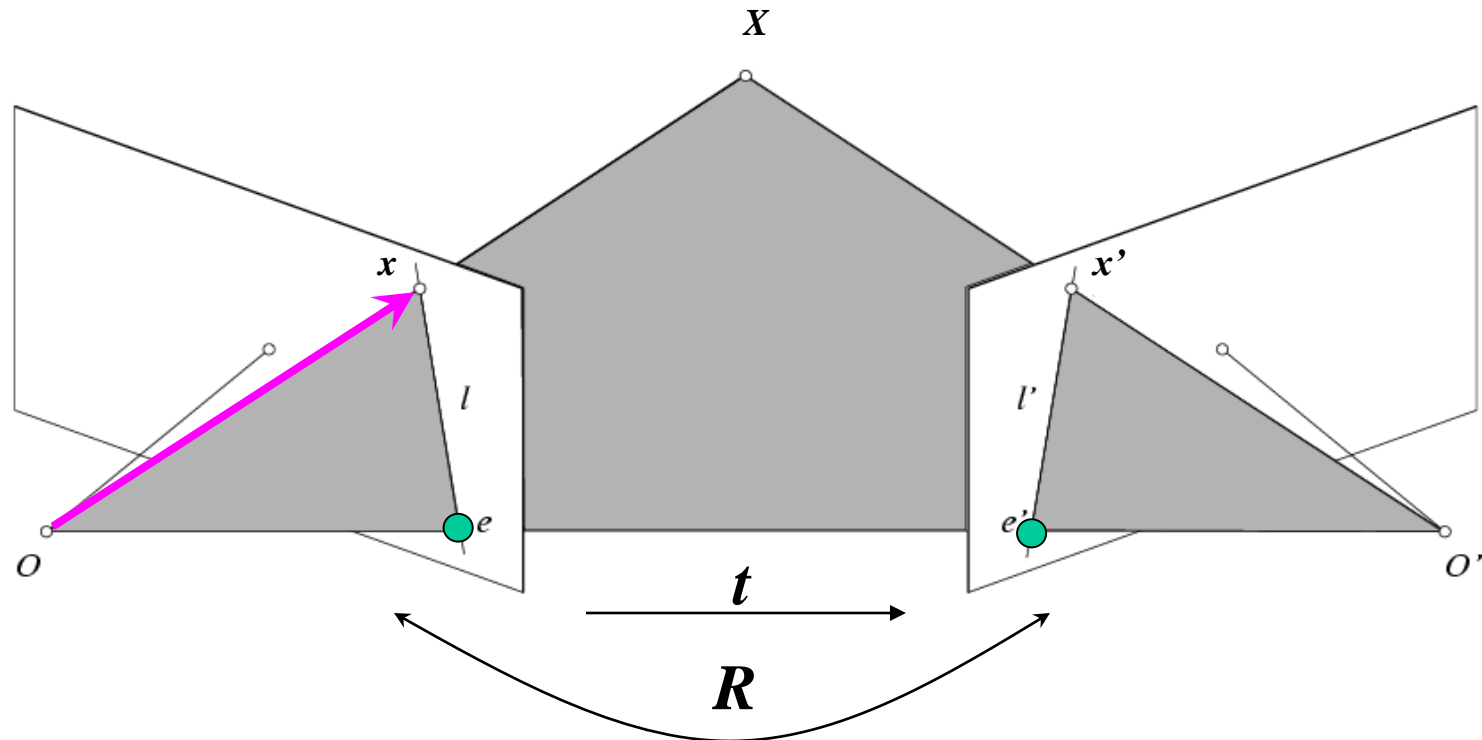


Camera matrix: $[I|0]$

$$X = (u, v, w, 1)^T$$

$$x = (u, v, w)^T$$

Epipolar constraint: Calibrated case



Camera matrix: $[I|0]$

$$X = (u, v, w, 1)^T$$

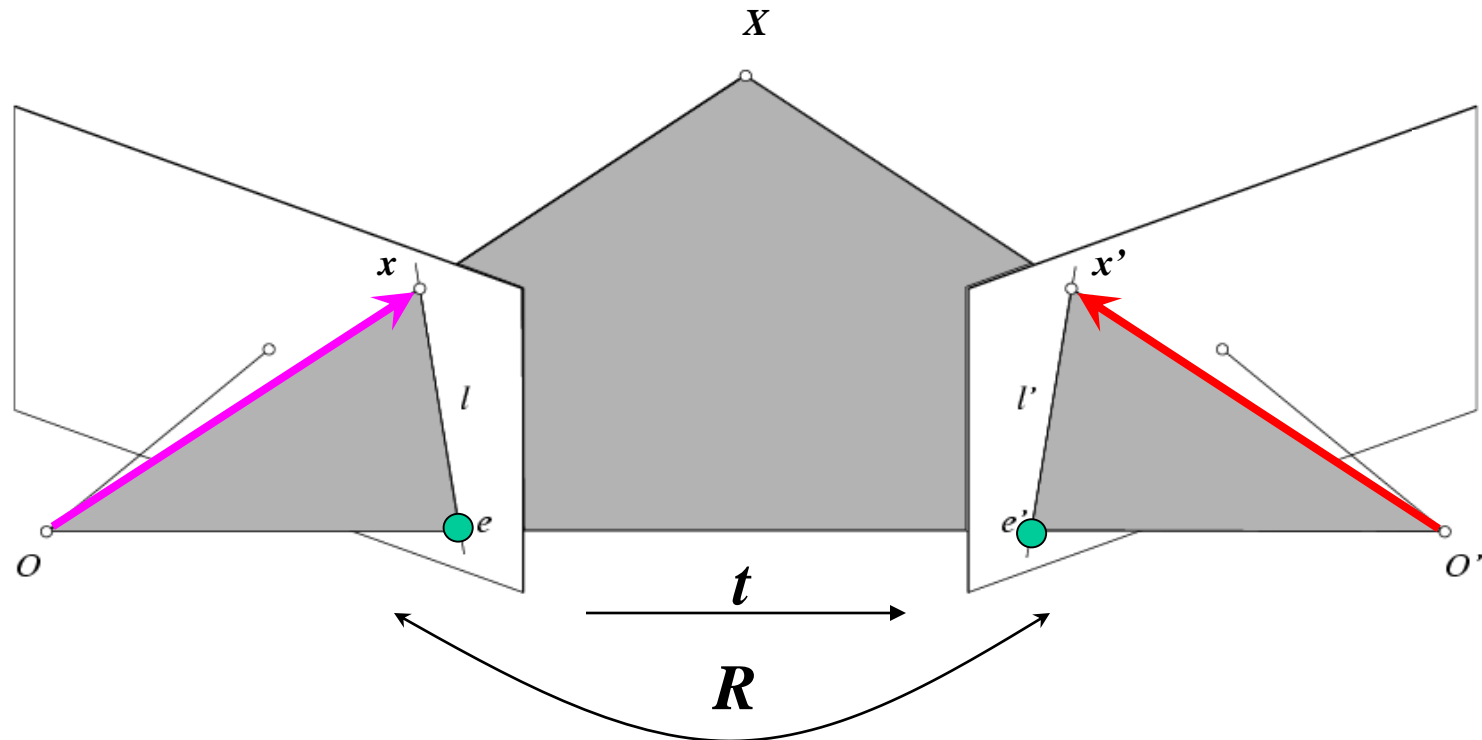
$$x = (u, v, w)^T$$

Camera matrix: $[R^T | -R^T t]$

Vector x' in second coord. system has coordinates

Rx' in the first one

Epipolar constraint: Calibrated case



Camera matrix: $[I|0]$

$$X = (u, v, w, 1)^T$$

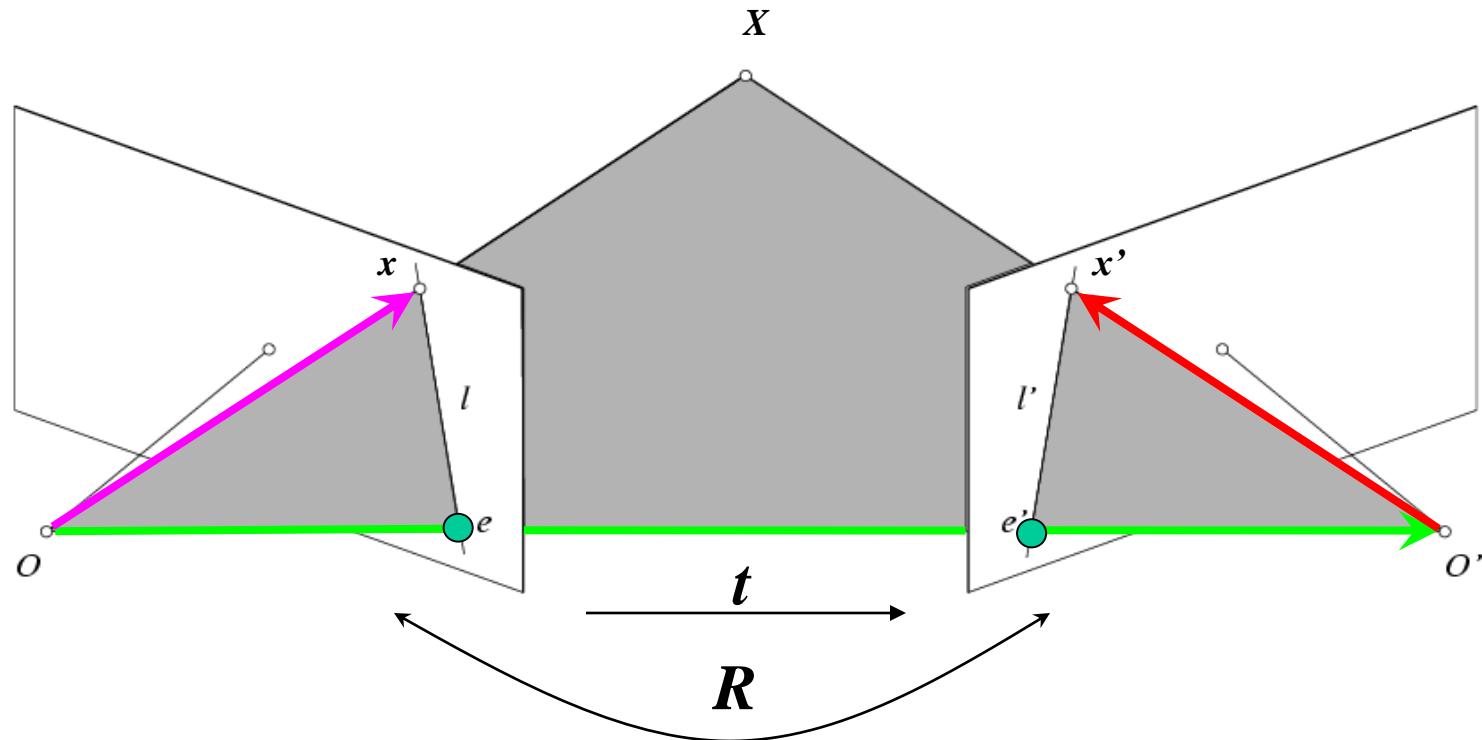
$$x = (u, v, w)^T$$

Camera matrix: $[R^T | -R^T t]$

Vector x' in second coord. system has coordinates

Rx' in the first one

Epipolar constraint: Calibrated case



Camera matrix: $[I|0]$

$$X = (u, v, w, 1)^T$$

$$x = (u, v, w)^T$$

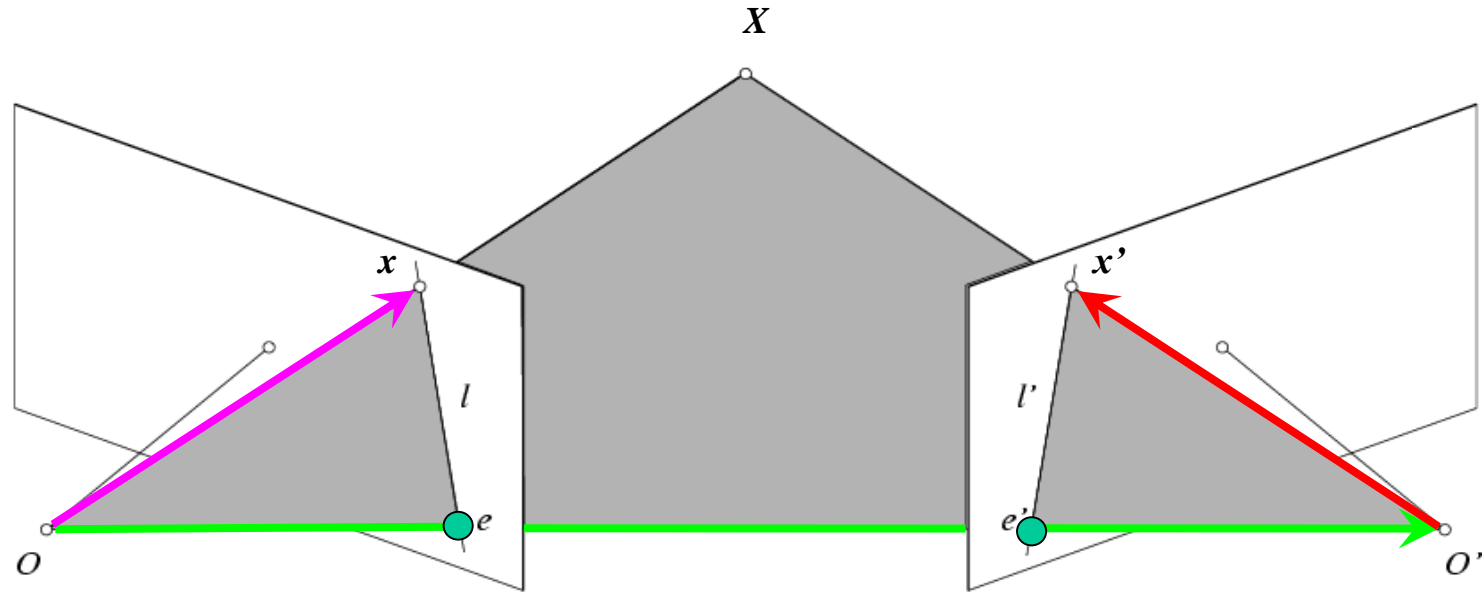
Camera matrix: $[R^T | -R^T t]$

Vector x' in second coord. system has coordinates

Rx' in the first one

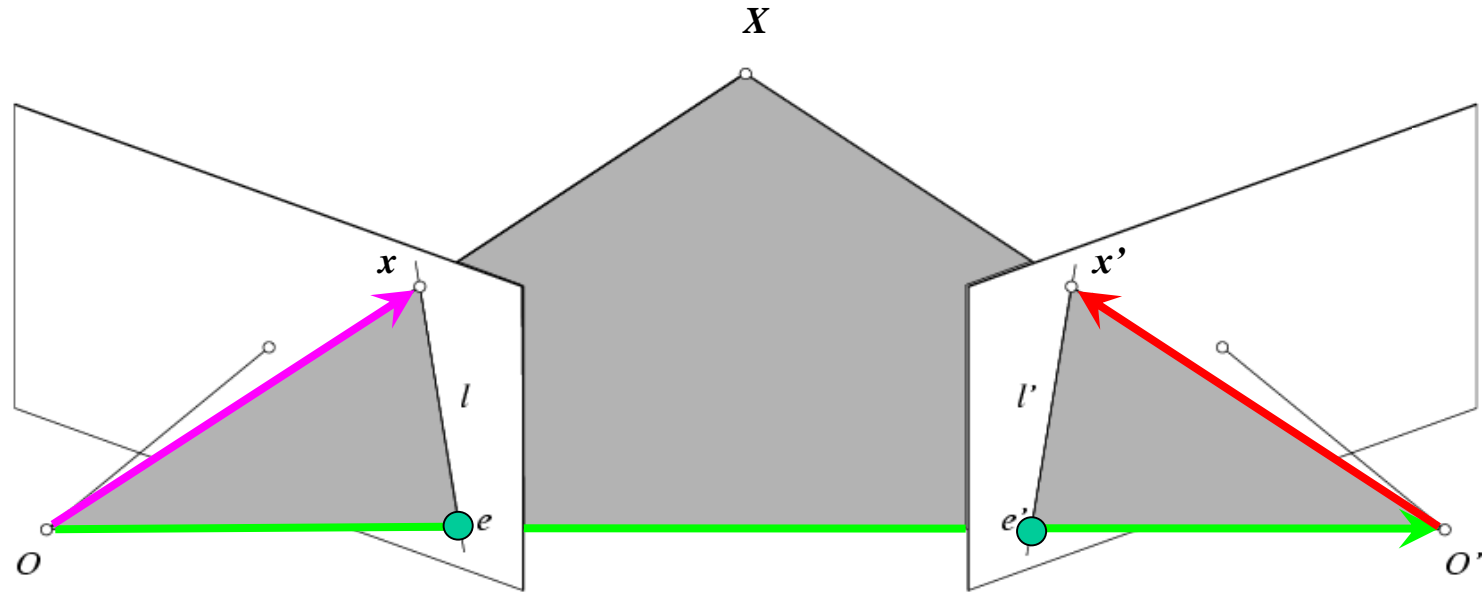
The vectors x , t , and Rx' are coplanar

Epipolar constraint: Calibrated case



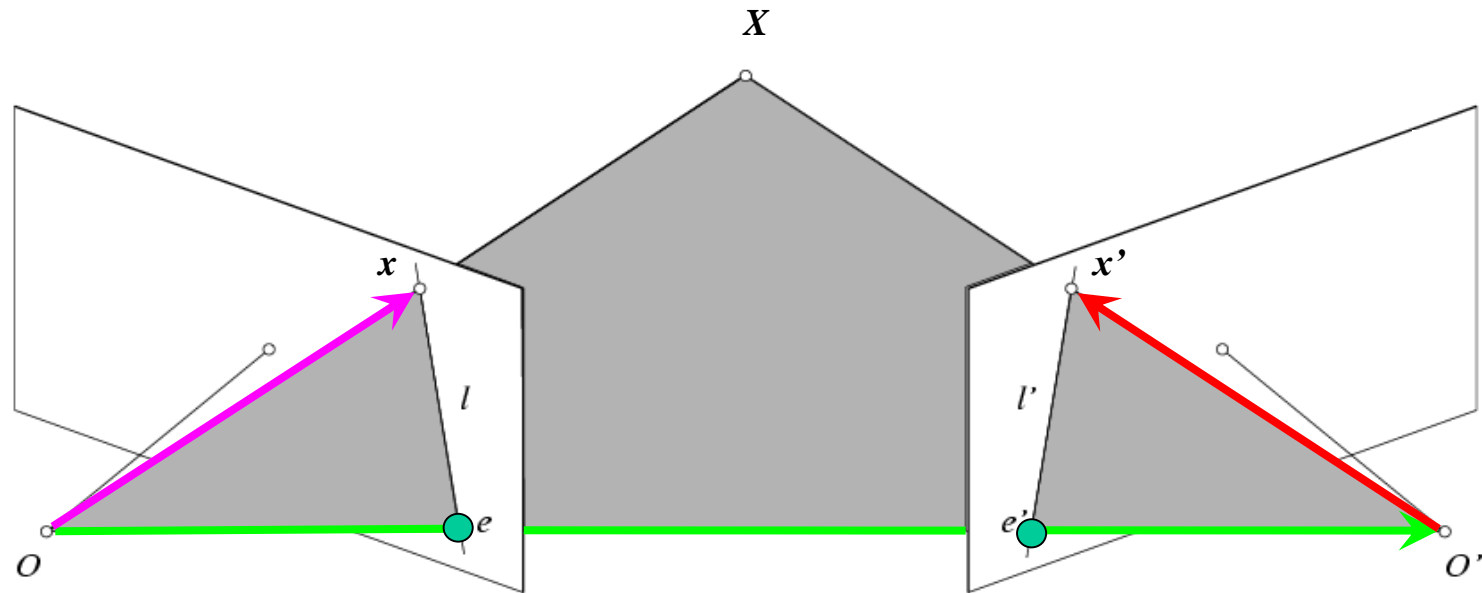
The vectors x , t , and Rx' are coplanar

Epipolar constraint: Calibrated case



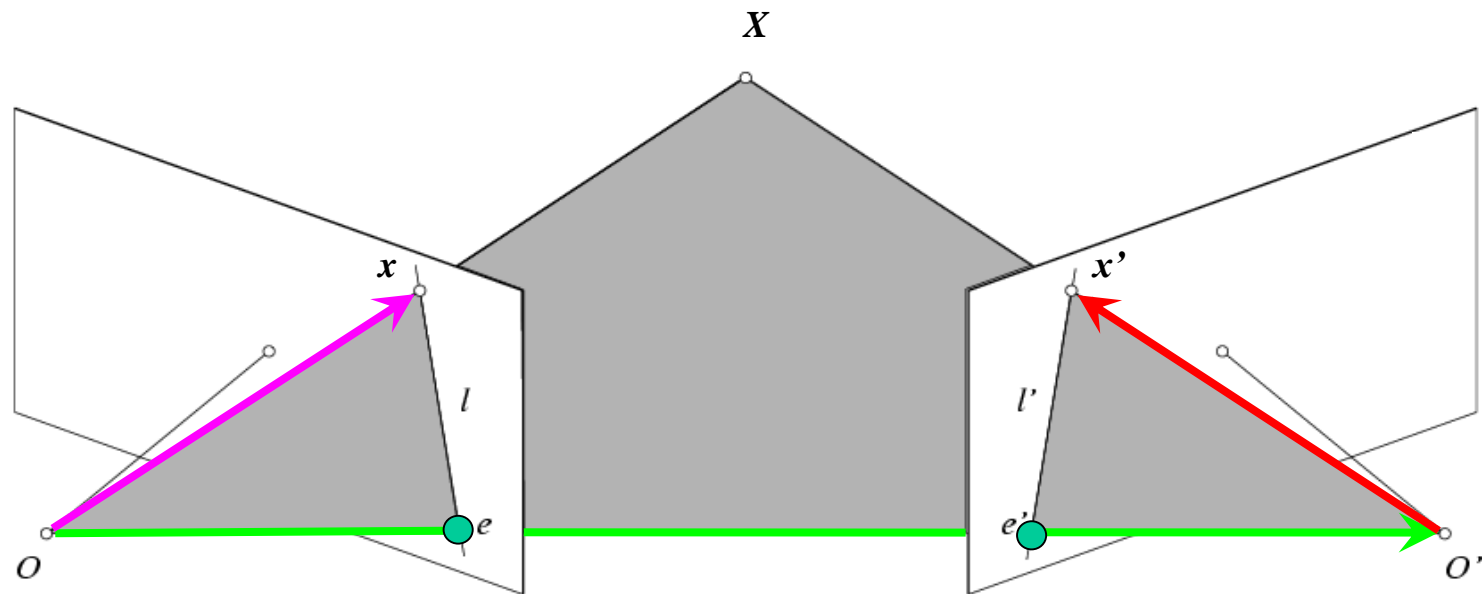
$$x \cdot [t \times (Rx')] = 0$$

Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_{\times}]R$$

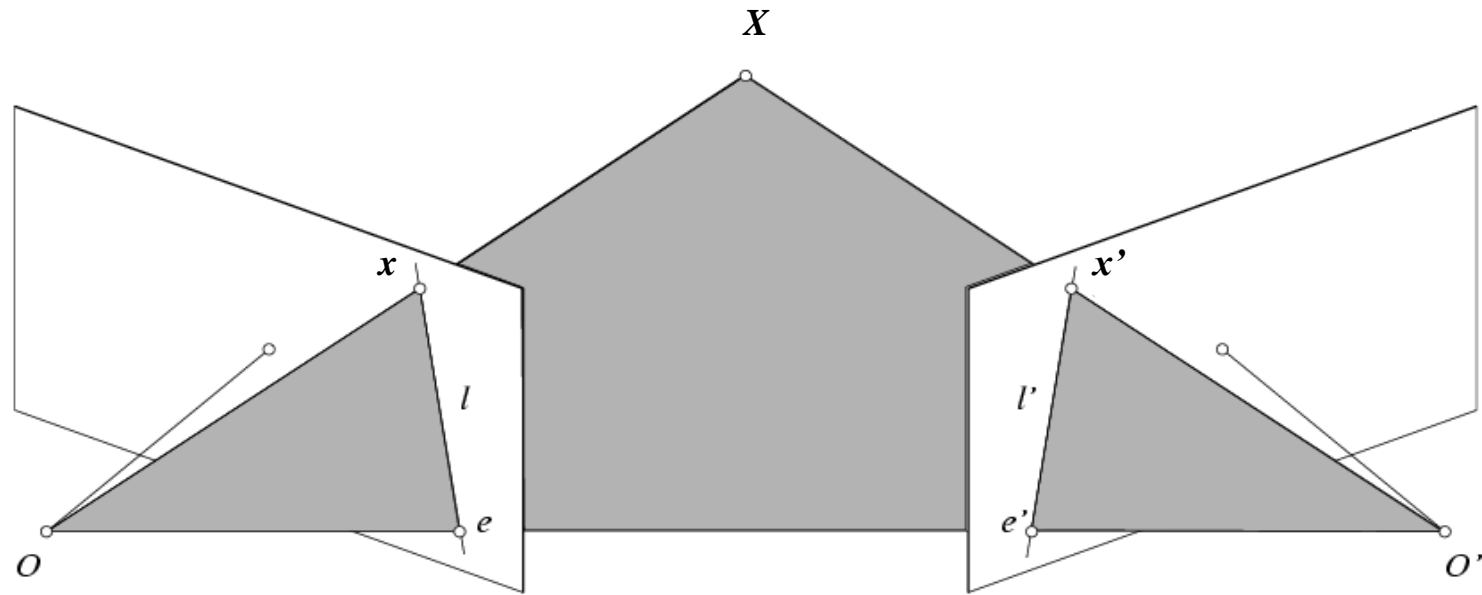
Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_{\times}] R$$

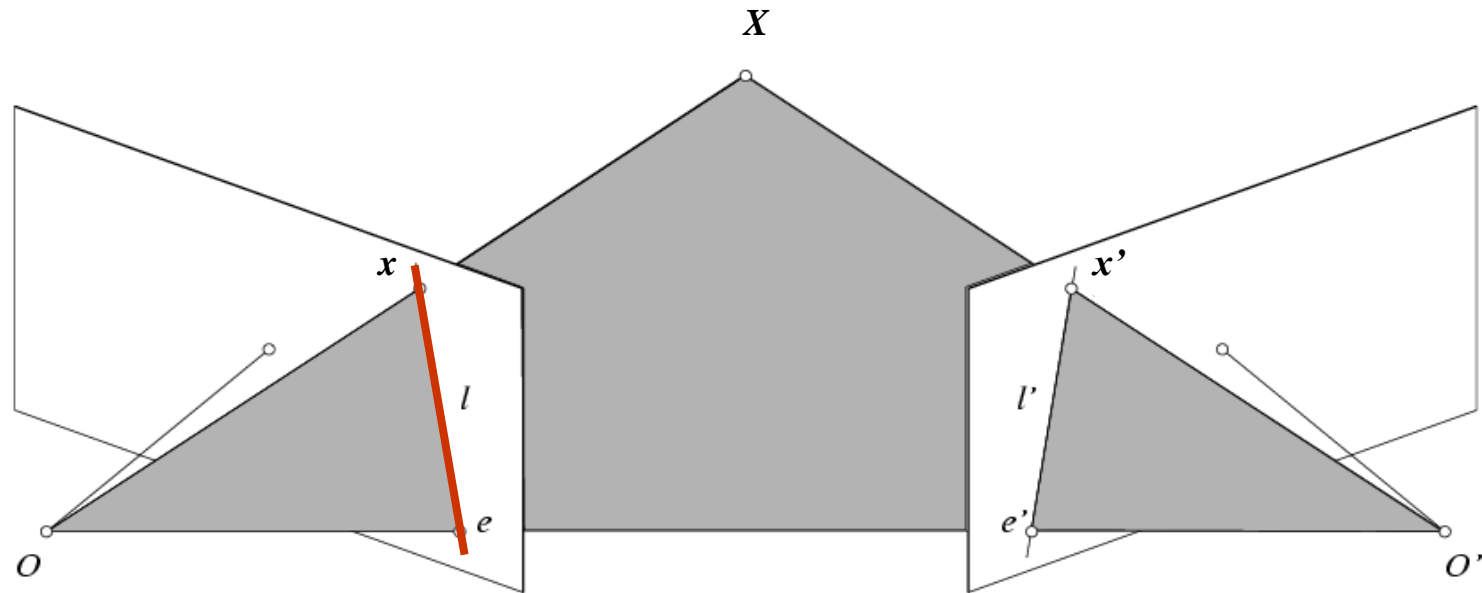
Essential Matrix
(Longuet-Higgins, 1981)

Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_{\times}]R$$

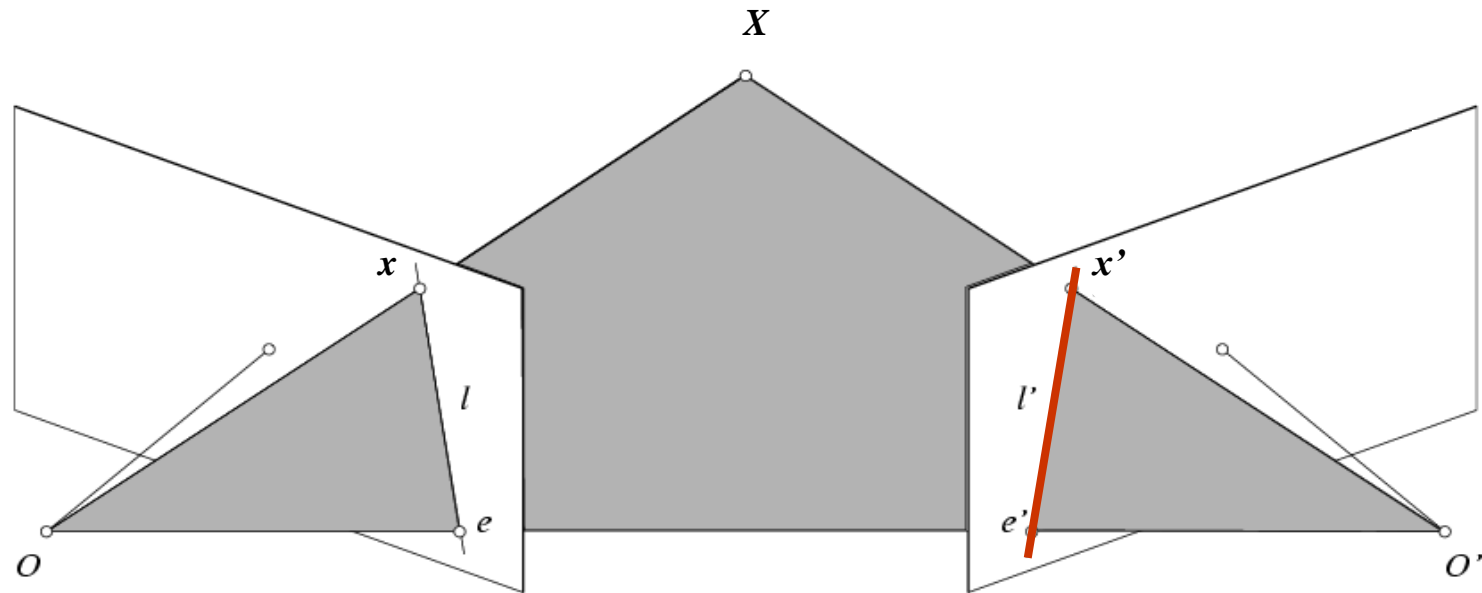
Epipolar constraint: Calibrated case



$$x \cdot [t \times (R x')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_x] R$$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)

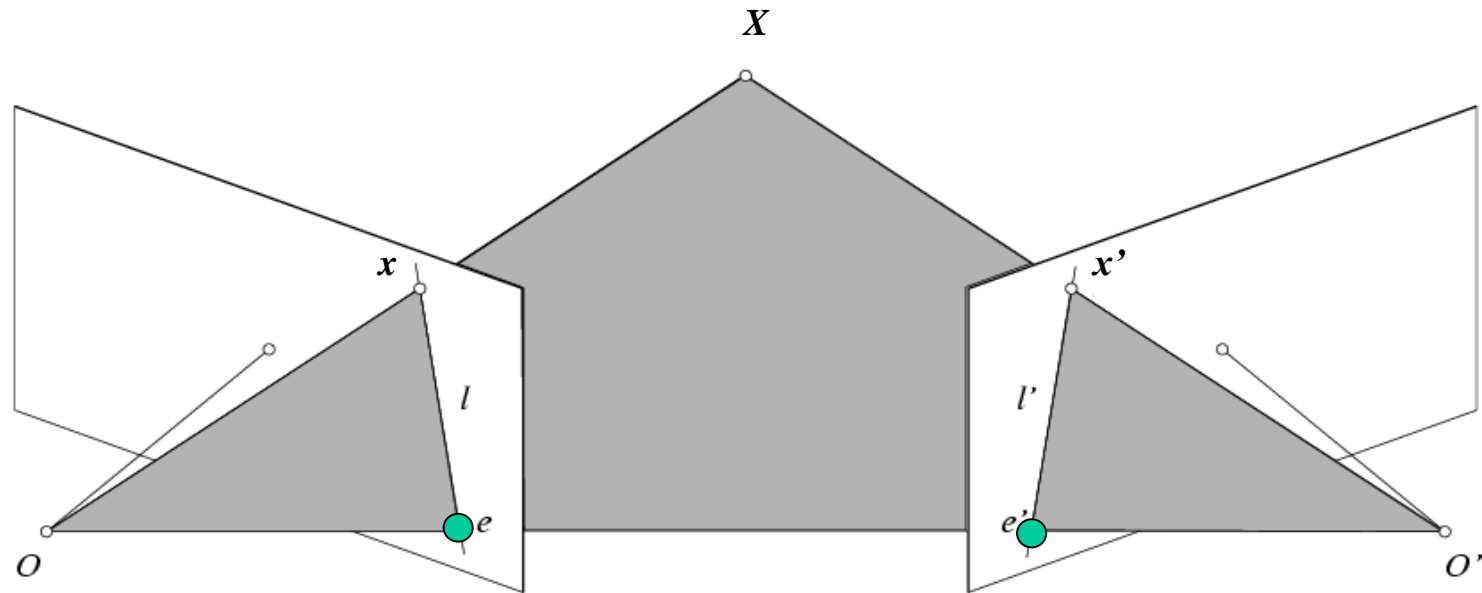
Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_x]R$$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)

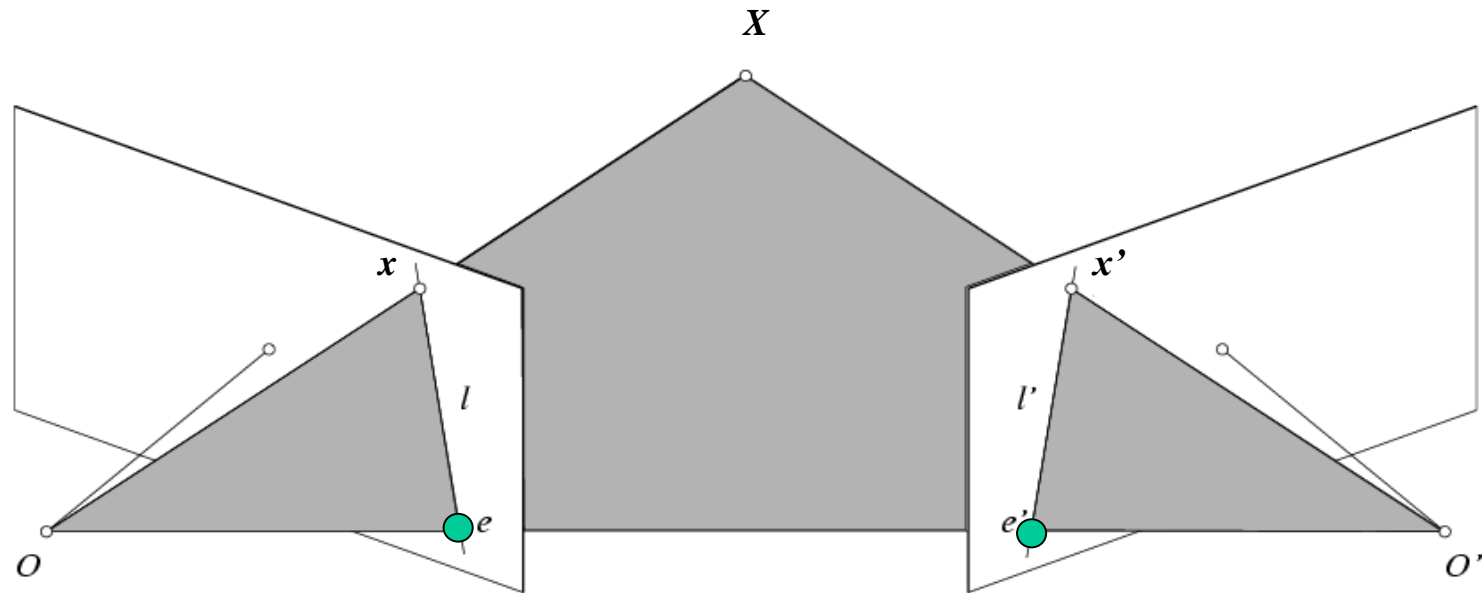
Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_{\times}] R$$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$

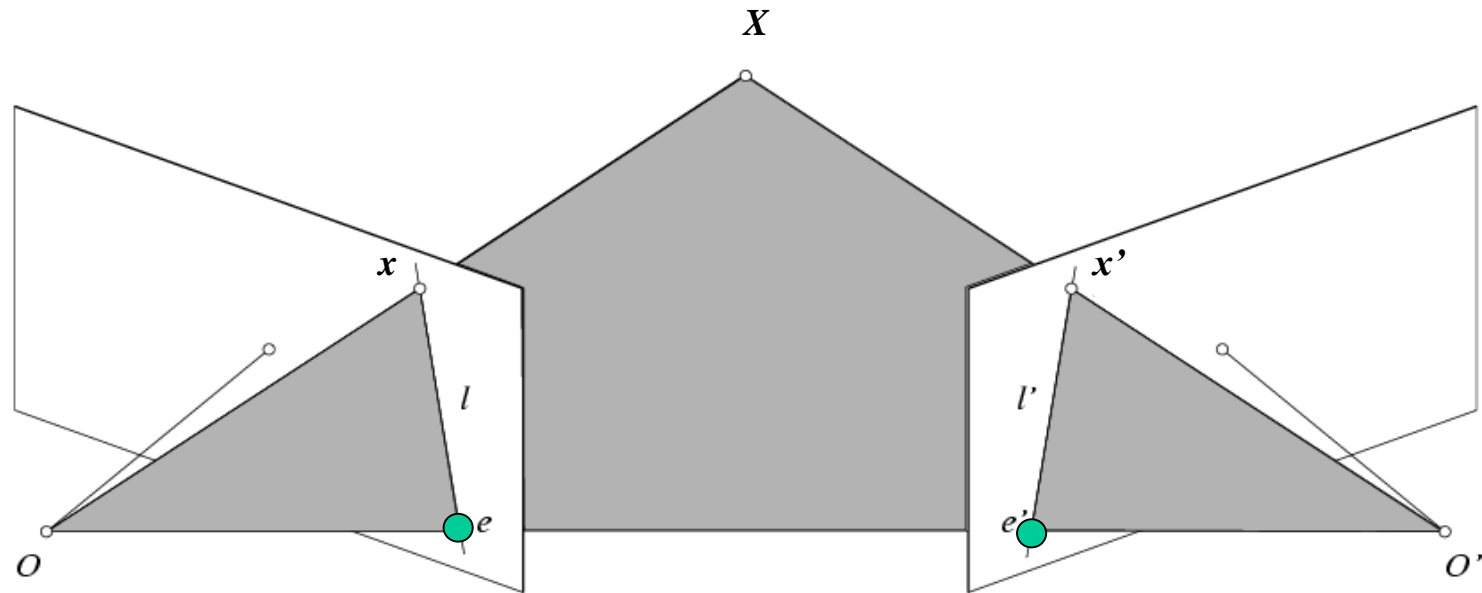
Epipolar constraint: Calibrated case



$$x \cdot [t \times (R x')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_{\times}] R$$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$
- E is singular (rank two)

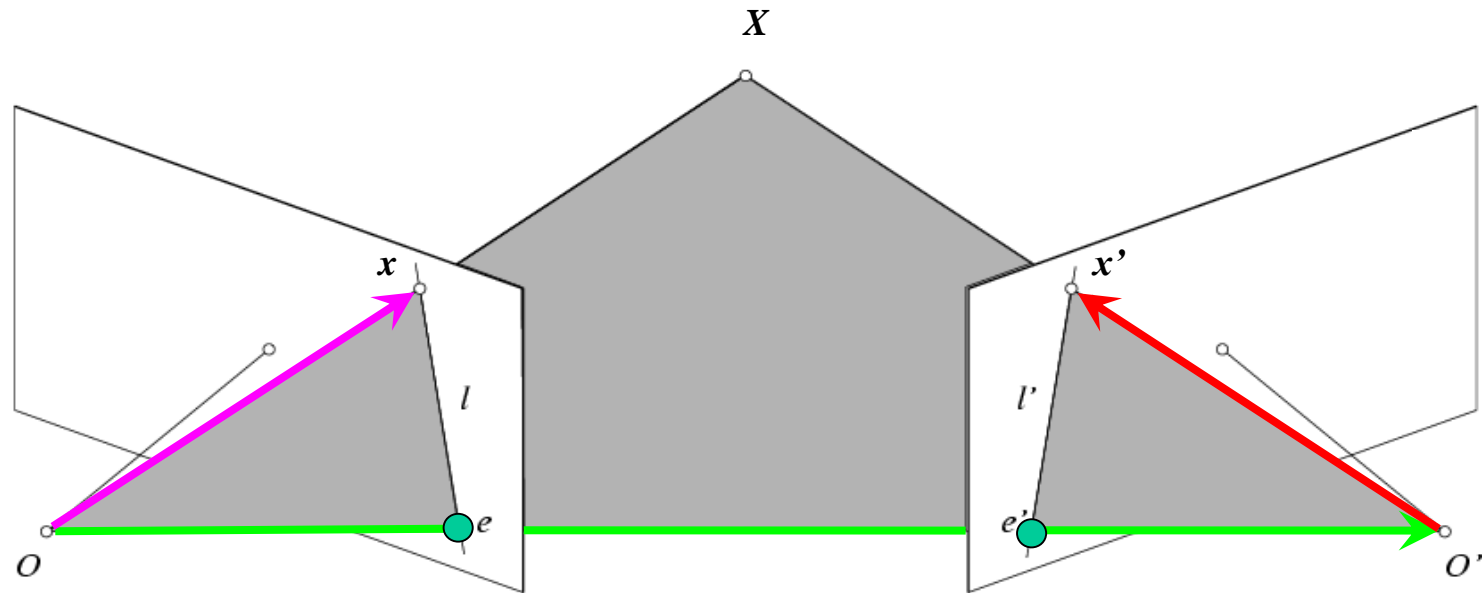
Epipolar constraint: Calibrated case



$$x \cdot [t \times (Rx')] = 0 \quad \longrightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_x]R$$

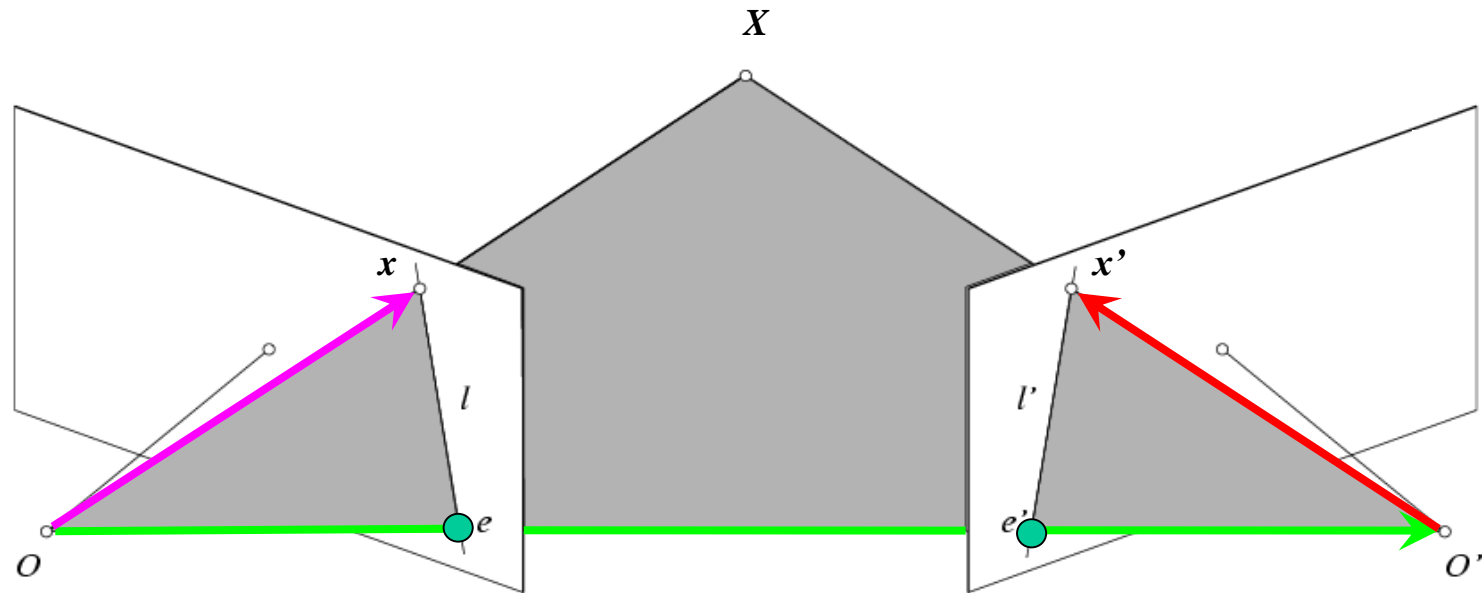
- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$
- E is singular (rank two)
- E has five degrees of freedom (up to scale)

Epipolar constraint: Uncalibrated case



- The calibration matrices K and K' of the two cameras are unknown

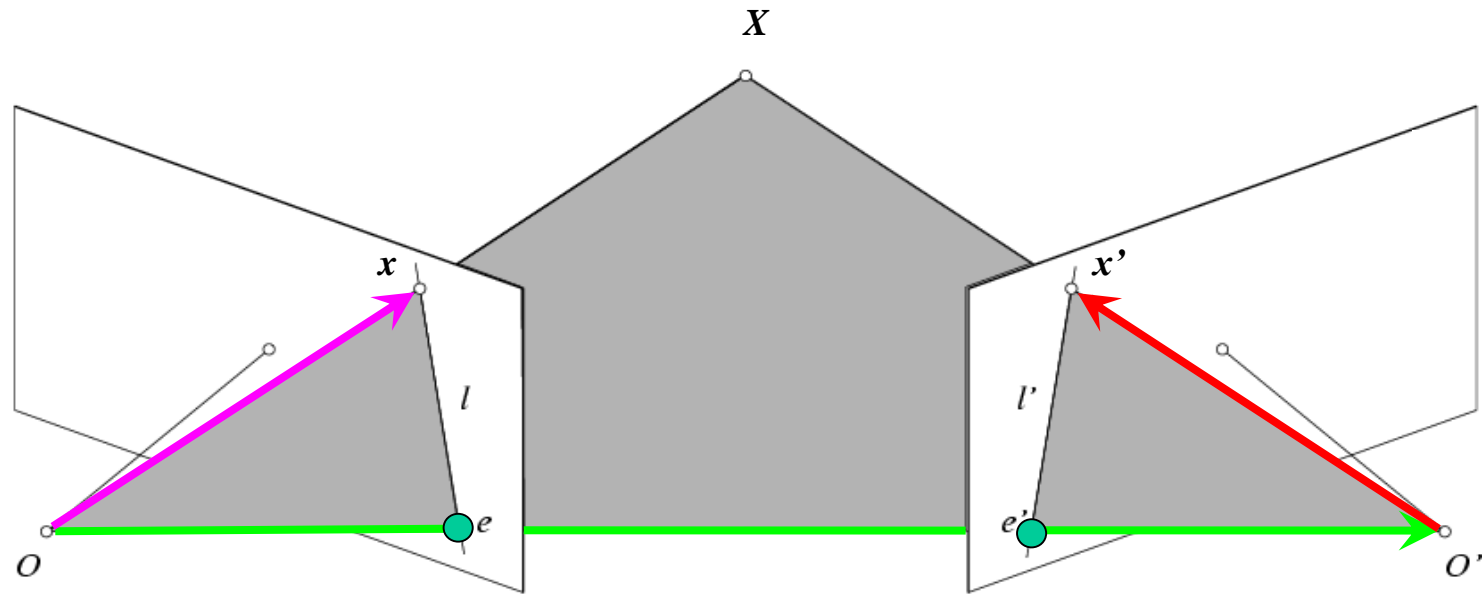
Epipolar constraint: Uncalibrated case



- The calibration matrices K and K' of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{x}^T E \hat{x}' = 0 \quad x = K \hat{x}, \quad x' = K' \hat{x}'$$

Epipolar constraint: Uncalibrated case

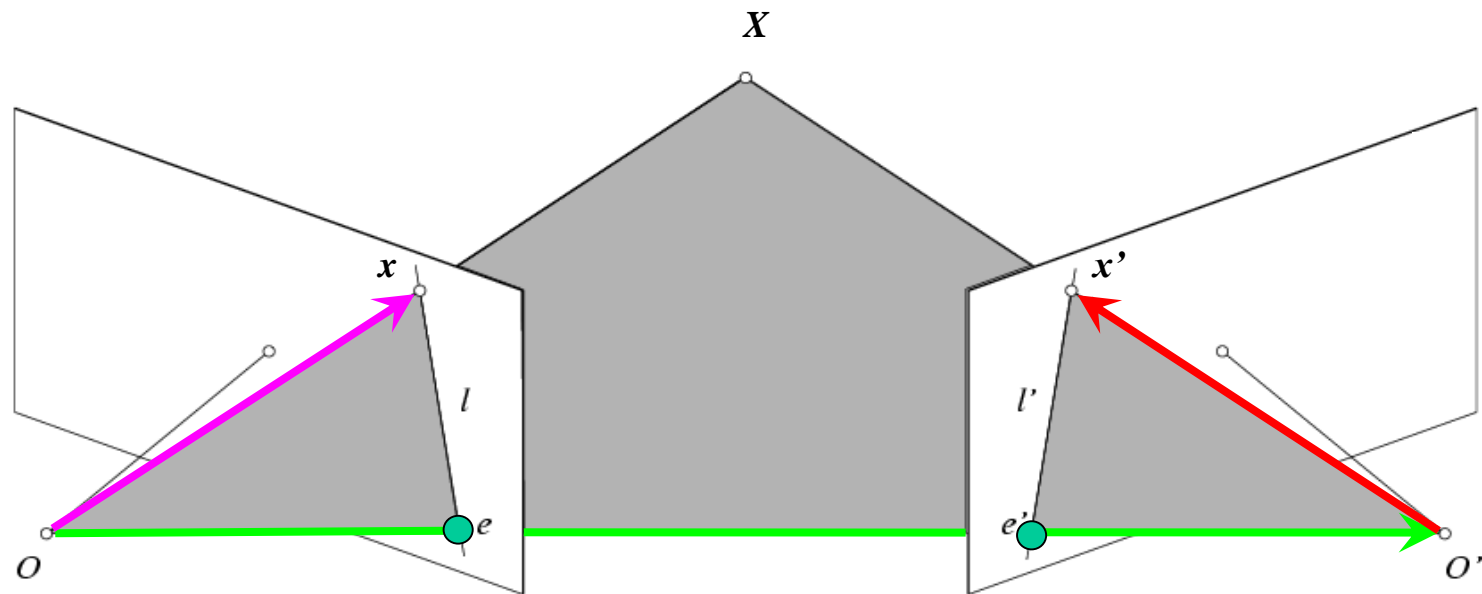


$$\hat{x}^T E \hat{x}' = 0$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

Epipolar constraint: Uncalibrated case

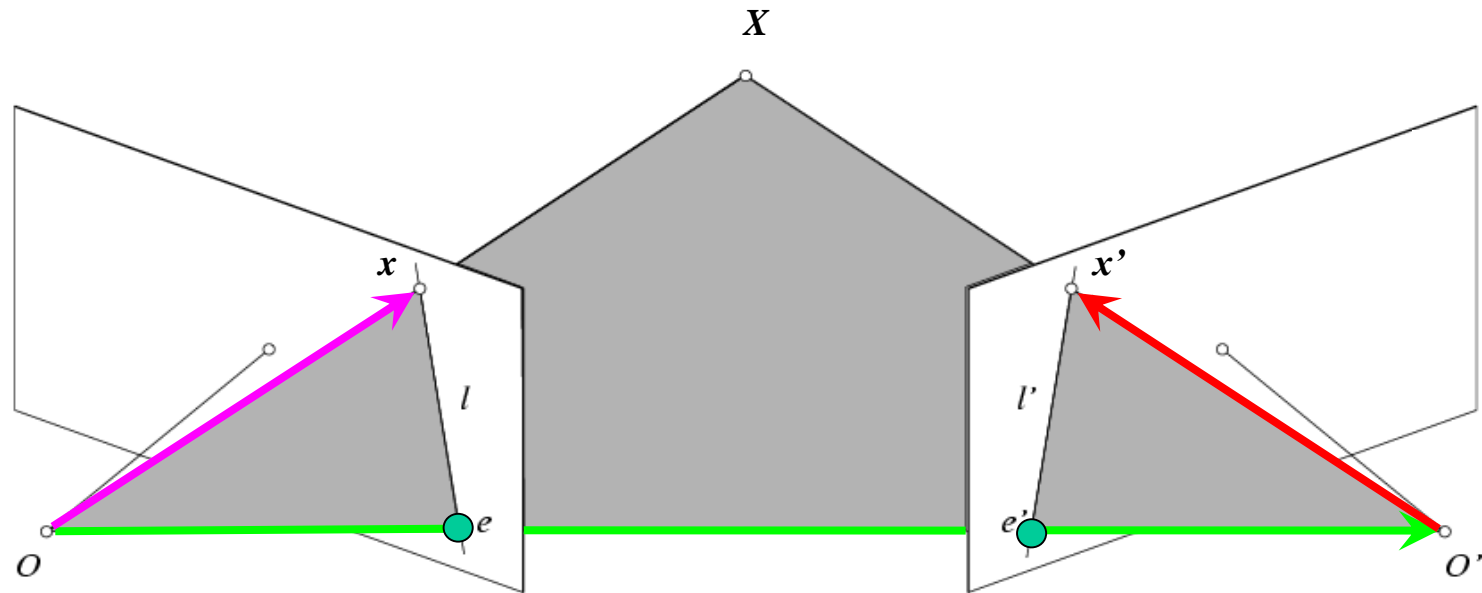


$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

Epipolar constraint: Uncalibrated case



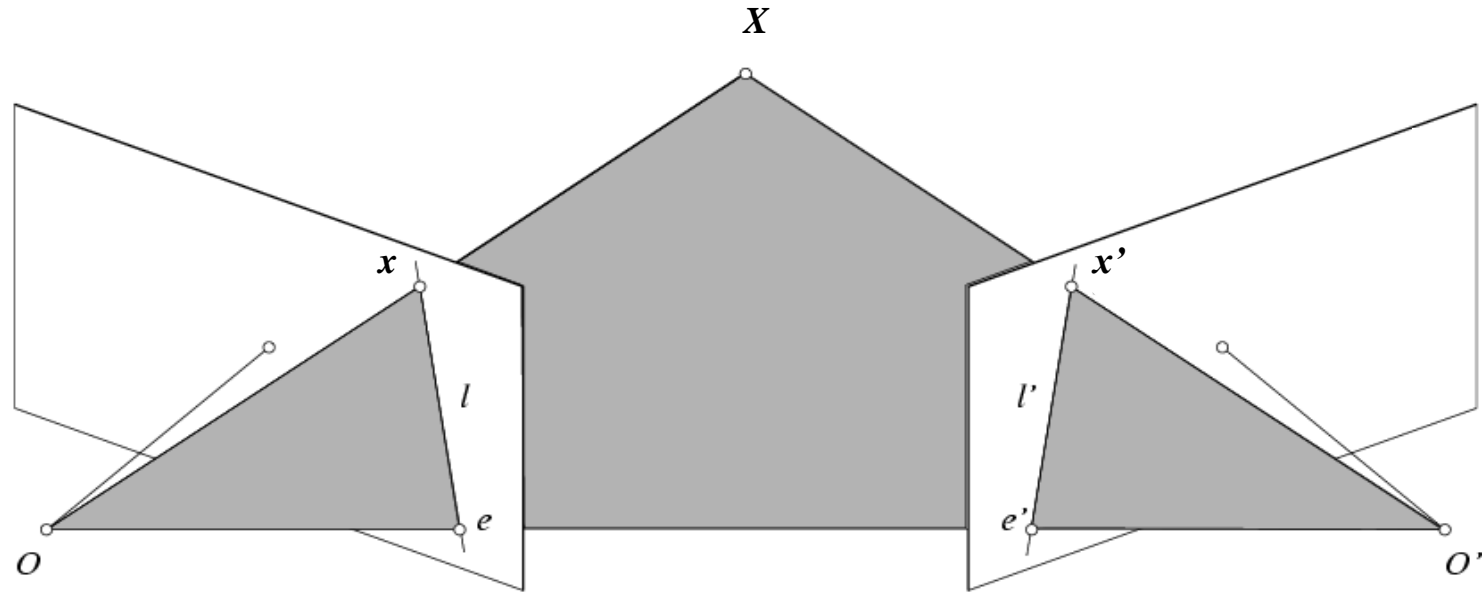
$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

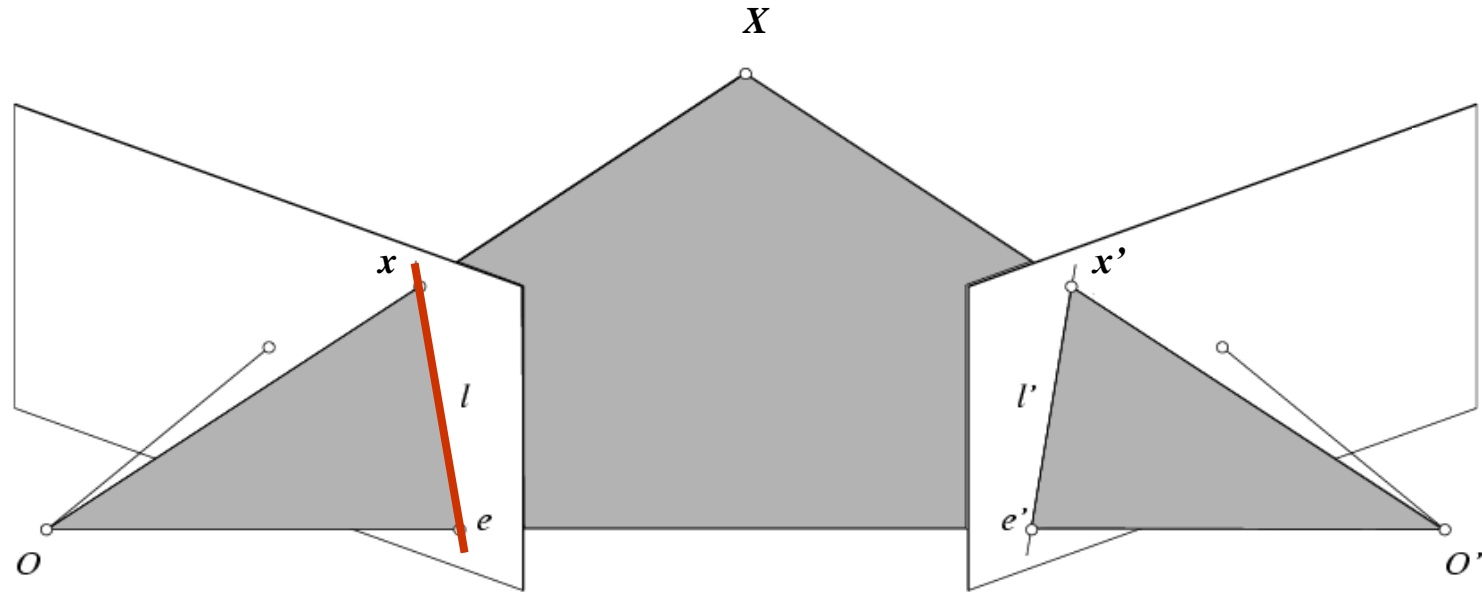
Fundamental Matrix
(Faugeras and Luong, 1992)

Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

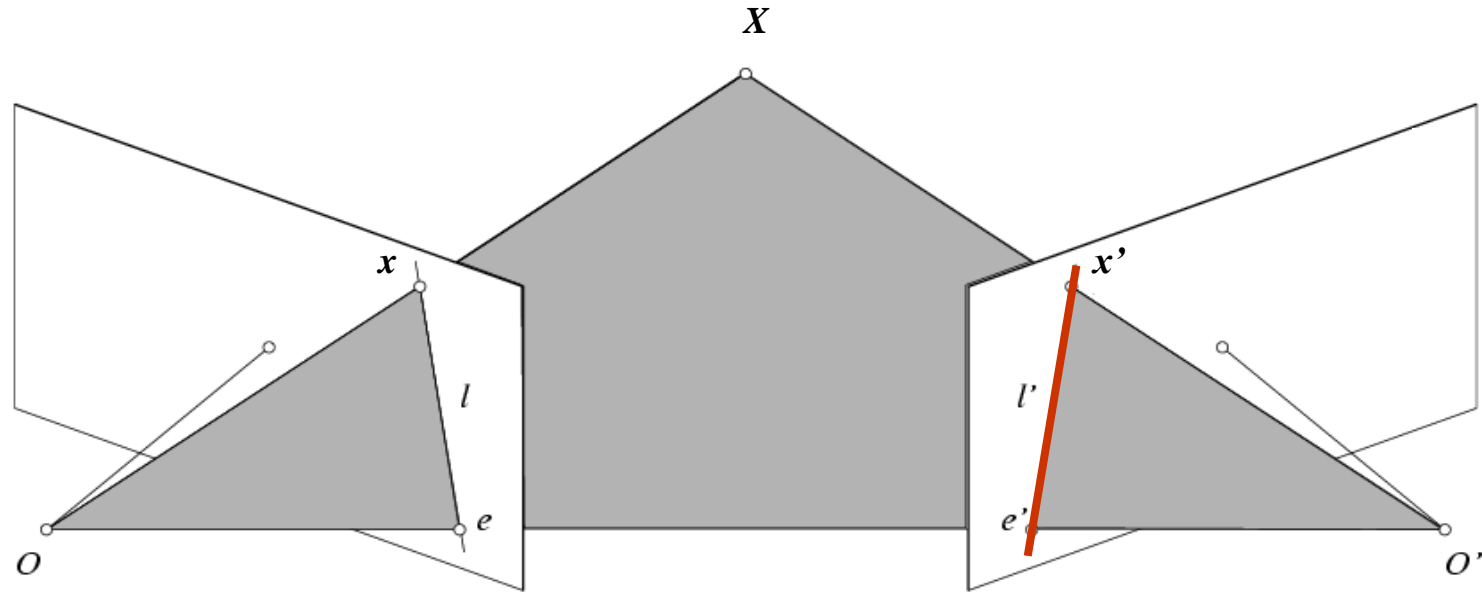
Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)

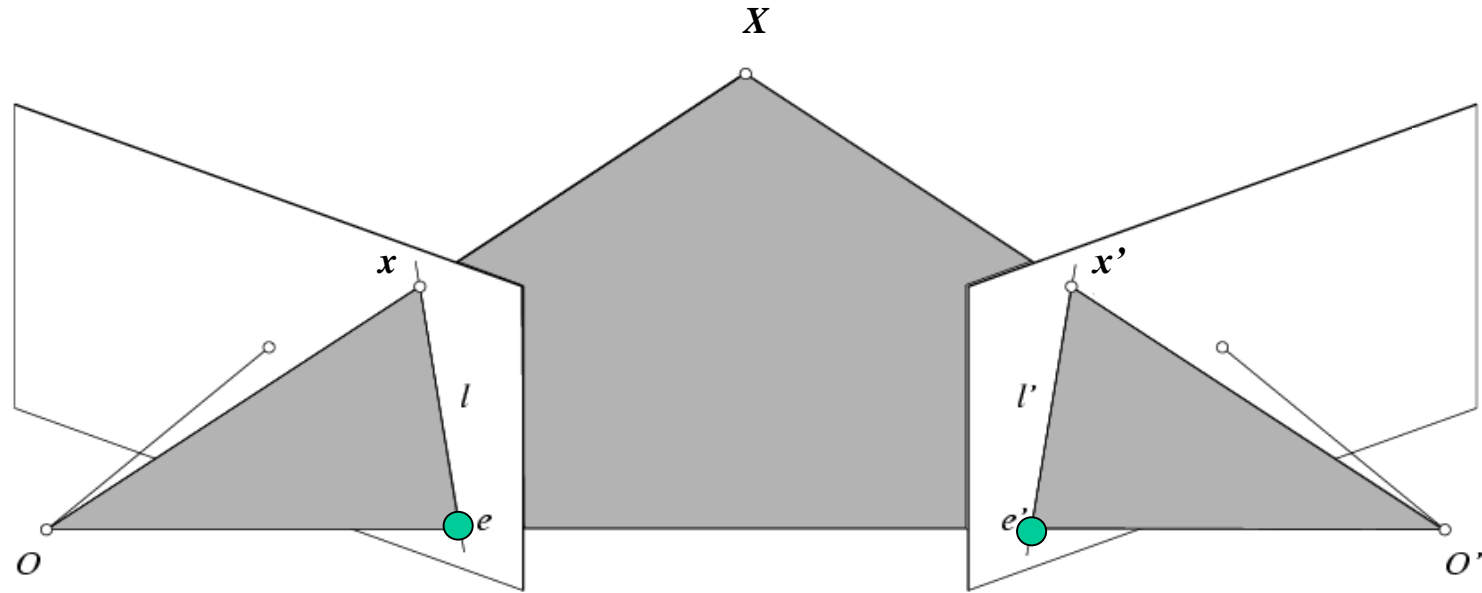
Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)

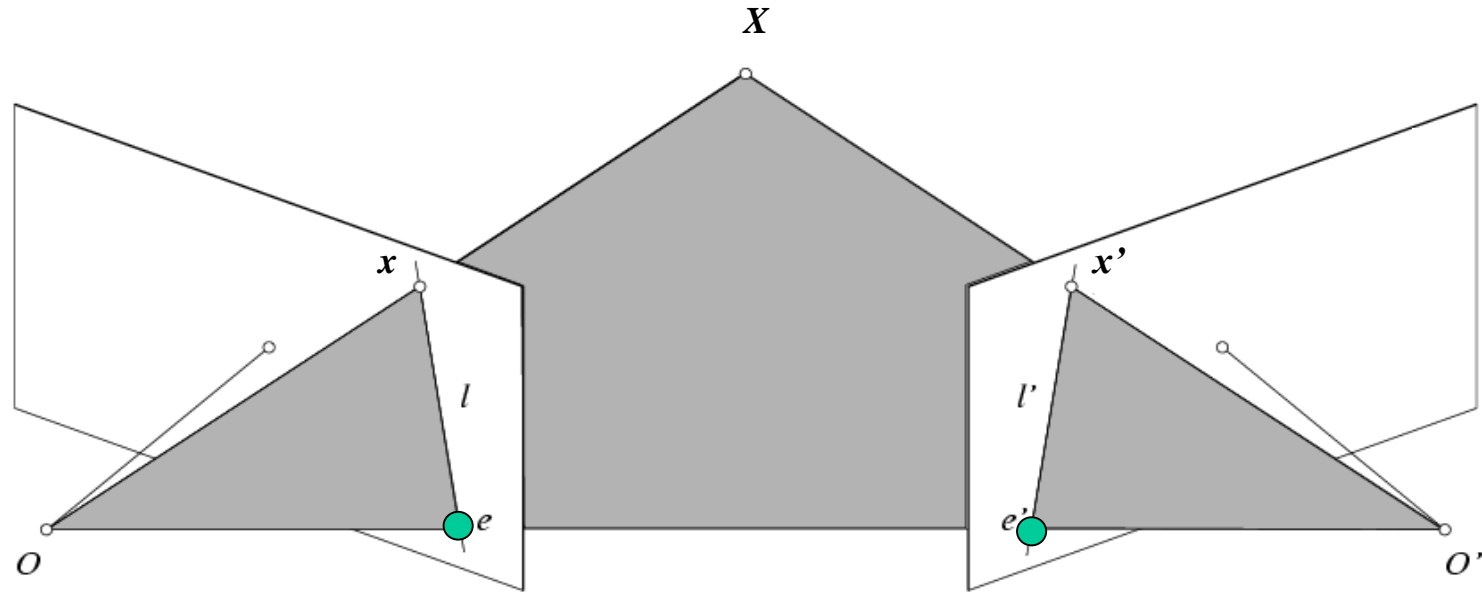
Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)
- $F e' = 0$ and $F^T e = 0$

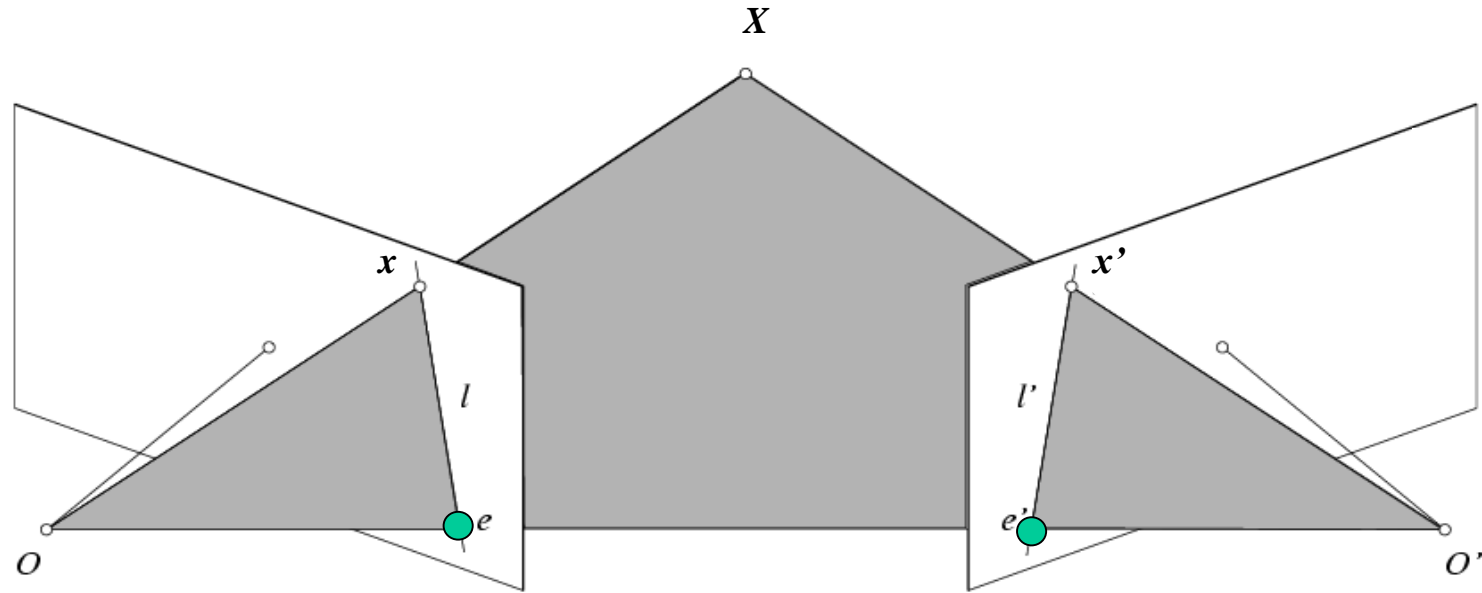
Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two)

Epipolar constraint: Uncalibrated case



$$\hat{x}^T E \hat{x}' = 0 \quad \longrightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two)
- F has seven degrees of freedom

The eight-point algorithm

$$\mathbf{x} = (u, v, 1)^T, \quad \mathbf{x}' = (u', v', 1)^T$$

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

The eight-point algorithm

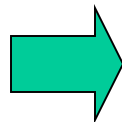
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$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \longrightarrow \quad (uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

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$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

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$$\begin{pmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 \\ u_3 u'_3 & u_3 v'_3 & u_3 & v_3 u'_3 & v_3 v'_3 & v_3 & u'_3 & v'_3 \\ u_4 u'_4 & u_4 v'_4 & u_4 & v_4 u'_4 & v_4 v'_4 & v_4 & u'_4 & v'_4 \\ u_5 u'_5 & u_5 v'_5 & u_5 & v_5 u'_5 & v_5 v'_5 & v_5 & u'_5 & v'_5 \\ u_6 u'_6 & u_6 v'_6 & u_6 & v_6 u'_6 & v_6 v'_6 & v_6 & u'_6 & v'_6 \\ u_7 u'_7 & u_7 v'_7 & u_7 & v_7 u'_7 & v_7 v'_7 & v_7 & u'_7 & v'_7 \\ u_8 u'_8 & u_8 v'_8 & u_8 & v_8 u'_8 & v_8 v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Minimize:

$$\sum_{i=1}^N (x_i^T F x'_i)^2$$

under the constraint

$$|F|^2 = 1$$

The eight-point algorithm

- Meaning of error $\sum_{i=1}^N (x_i^T F x'_i)^2$:

sum of Euclidean distances between points x_i and epipolar lines Fx'_i (or points x'_i and epipolar lines $F^T x_i$) multiplied by a scale factor

- Nonlinear approach: minimize

$$\sum_{i=1}^N \left[d^2(x_i, F x'_i) + d^2(x'_i, F^T x_i) \right]$$

Problem with eight-point algorithm

$$\begin{pmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 \\ u_3 u'_3 & u_3 v'_3 & u_3 & v_3 u'_3 & v_3 v'_3 & v_3 & u'_3 & v'_3 \\ u_4 u'_4 & u_4 v'_4 & u_4 & v_4 u'_4 & v_4 v'_4 & v_4 & u'_4 & v'_4 \\ u_5 u'_5 & u_5 v'_5 & u_5 & v_5 u'_5 & v_5 v'_5 & v_5 & u'_5 & v'_5 \\ u_6 u'_6 & u_6 v'_6 & u_6 & v_6 u'_6 & v_6 v'_6 & v_6 & u'_6 & v'_6 \\ u_7 u'_7 & u_7 v'_7 & u_7 & v_7 u'_7 & v_7 v'_7 & v_7 & u'_7 & v'_7 \\ u_8 u'_8 & u_8 v'_8 & u_8 & v_8 u'_8 & v_8 v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Problem with eight-point algorithm

| | | | | | | | |
|-----------|-----------|--------|-----------|-----------|--------|--------|--------|
| 250906.36 | 183269.57 | 921.81 | 200931.10 | 146766.13 | 738.21 | 272.19 | 198.81 |
| 2692.28 | 131633.03 | 176.27 | 6196.73 | 302975.59 | 405.71 | 15.27 | 746.79 |
| 416374.23 | 871684.30 | 935.47 | 408110.89 | 854384.92 | 916.90 | 445.10 | 931.81 |
| 191183.60 | 171759.40 | 410.27 | 416435.62 | 374125.90 | 893.65 | 465.99 | 418.65 |
| 48988.86 | 30401.76 | 57.89 | 298604.57 | 185309.58 | 352.87 | 846.22 | 525.15 |
| 164786.04 | 546559.67 | 813.17 | 1998.37 | 6628.15 | 9.86 | 202.65 | 672.14 |
| 116407.01 | 2727.75 | 138.89 | 169941.27 | 3982.21 | 202.77 | 838.12 | 19.64 |
| 135384.58 | 75411.13 | 198.72 | 411350.03 | 229127.78 | 603.79 | 681.28 | 379.48 |

$$\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Poor numerical conditioning

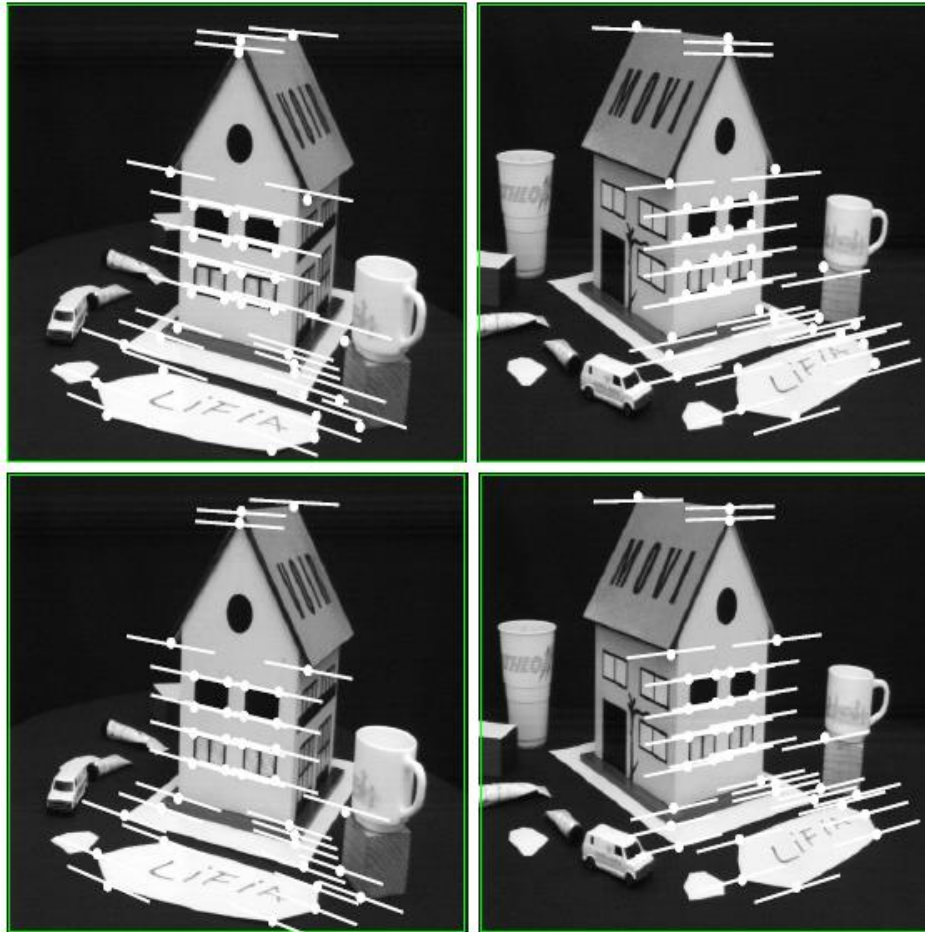
Can be fixed by rescaling the data

The normalized eight-point algorithm

(Hartley, 1995)

- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute F from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of F and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if T and T' are the normalizing transformations in the two images, then the fundamental matrix in original coordinates is $T^T F T'$

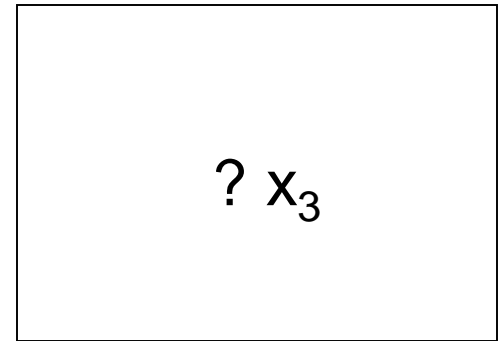
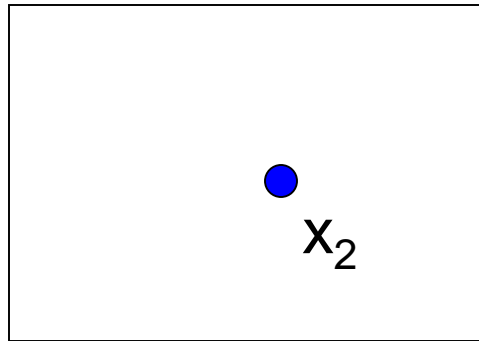
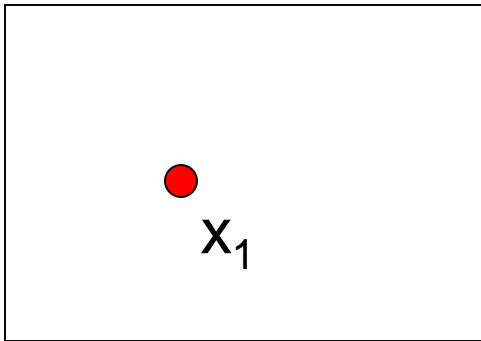
Comparison of estimation algorithms



| | 8-point | Normalized 8-point | Nonlinear least squares |
|-------------|-------------|--------------------|-------------------------|
| Av. Dist. 1 | 2.33 pixels | 0.92 pixel | 0.86 pixel |
| Av. Dist. 2 | 2.18 pixels | 0.85 pixel | 0.80 pixel |

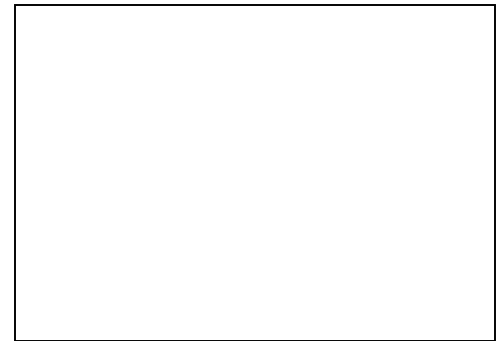
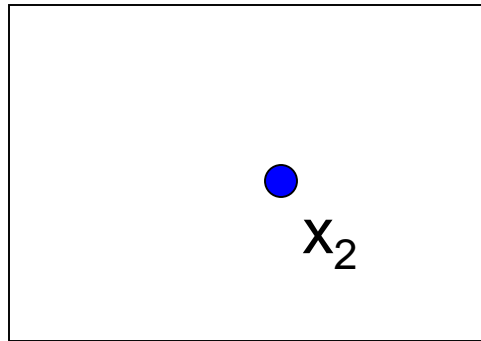
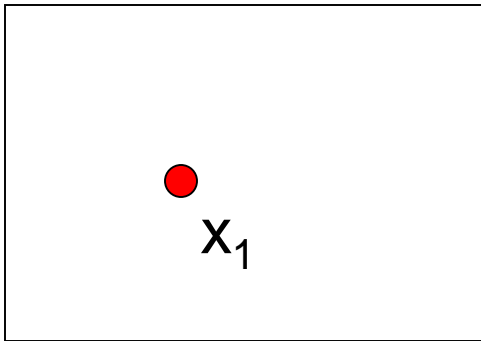
Epipolar transfer

- Assume the epipolar geometry is known
- Given projections of the same point in two images, how can we compute the projection of that point in a third image?



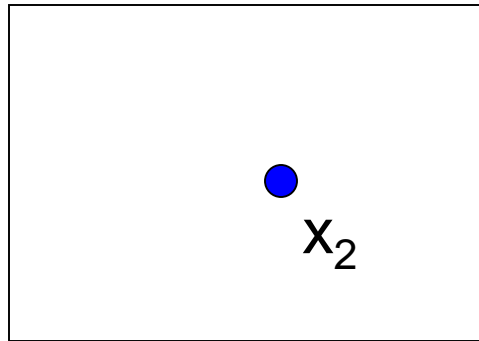
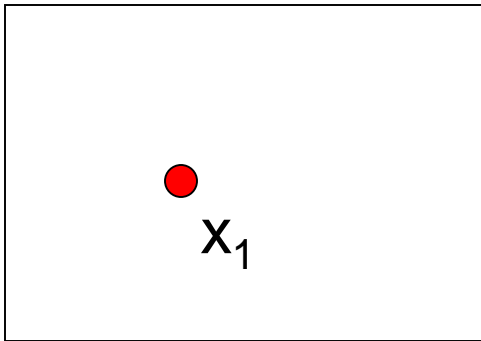
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Epipolar transfer

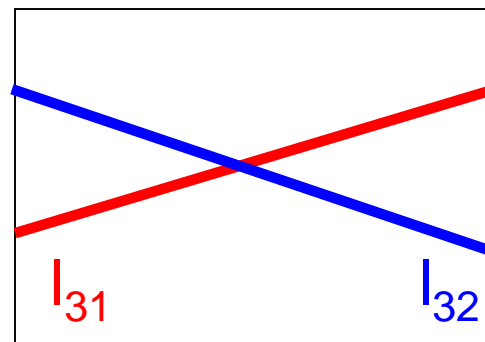
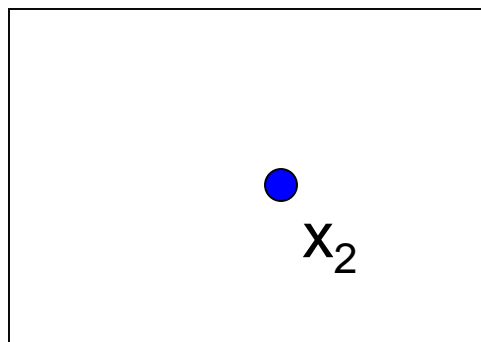
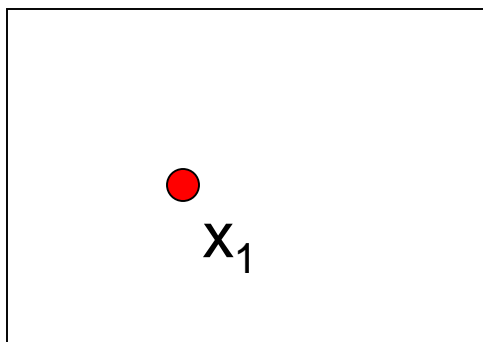
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$$l_{31} = F_{13}^T X_1$$

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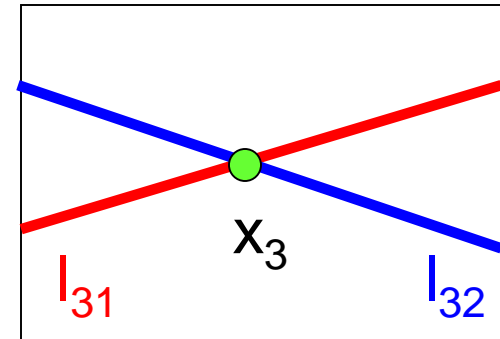
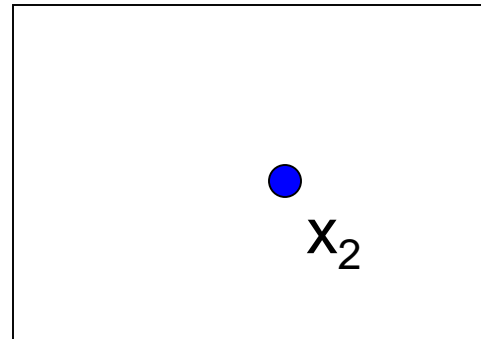
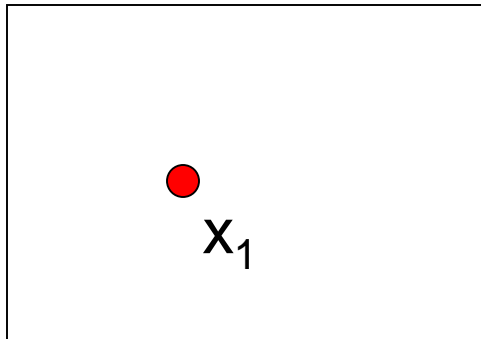


$$l_{31} = F_{13}^T X_1$$

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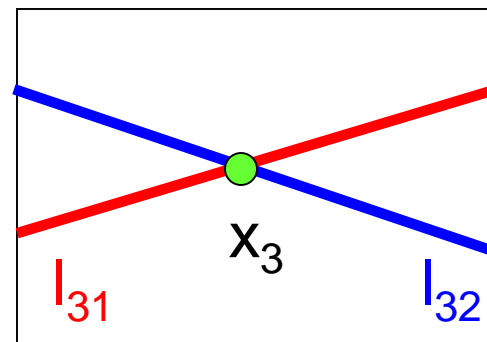
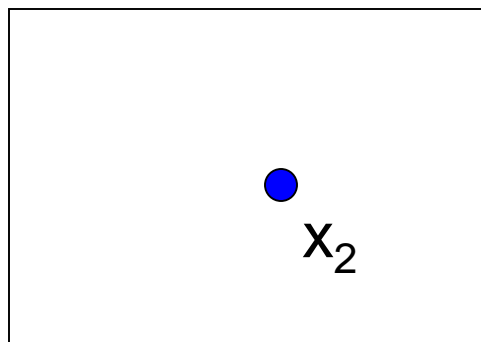
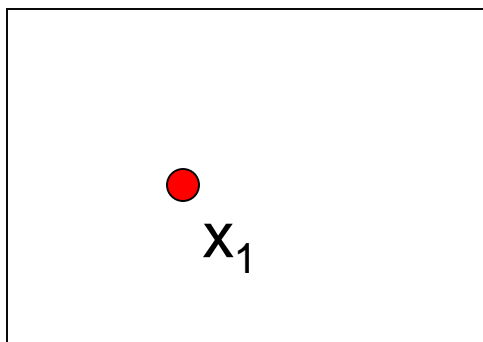


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When does epipolar transfer fail?