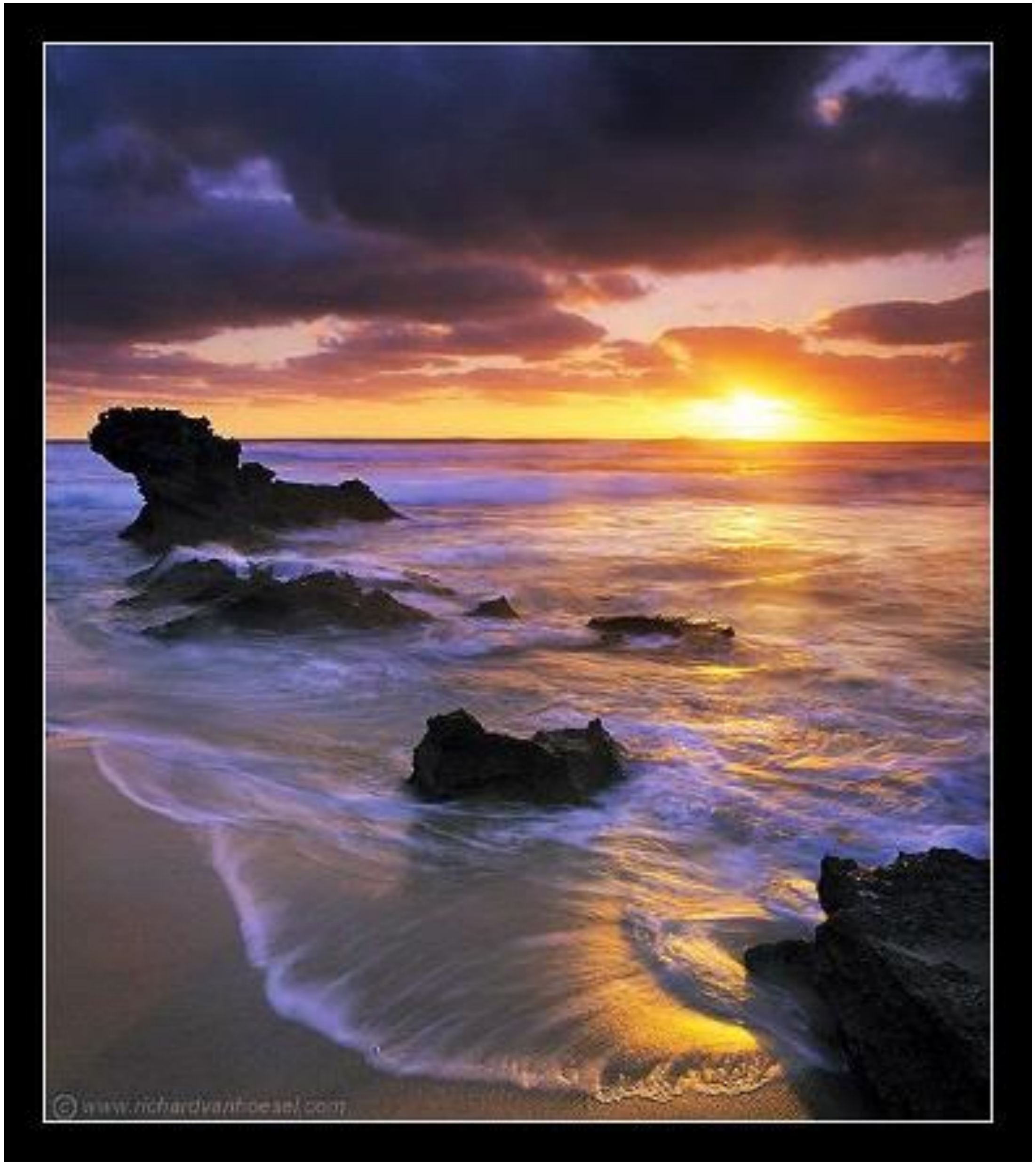
Capturing light



Source: A. Efros

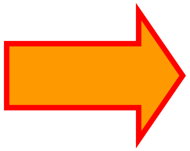
Review

- Pinhole projection models
  - What are vanishing points and vanishing lines?
  - What is orthographic projection?
  - How can we approximate orthographic projection?
- Lenses
  - Why do we need lenses?
  - What controls depth of field?
  - What controls field of view?
  - What are some kinds of lens aberrations?
- Digital cameras
  - What are the two major types of sensor technologies?
  - What are the different types of color sensors?

Aside: Early color photography

Sergey Prokudin-Gorsky (1863-1944)

Photographs of the Russian empire (1909-1916)



http://en.wikipedia.org/wiki/Sergei_Mikhailovich_Prokudin-Gorskii

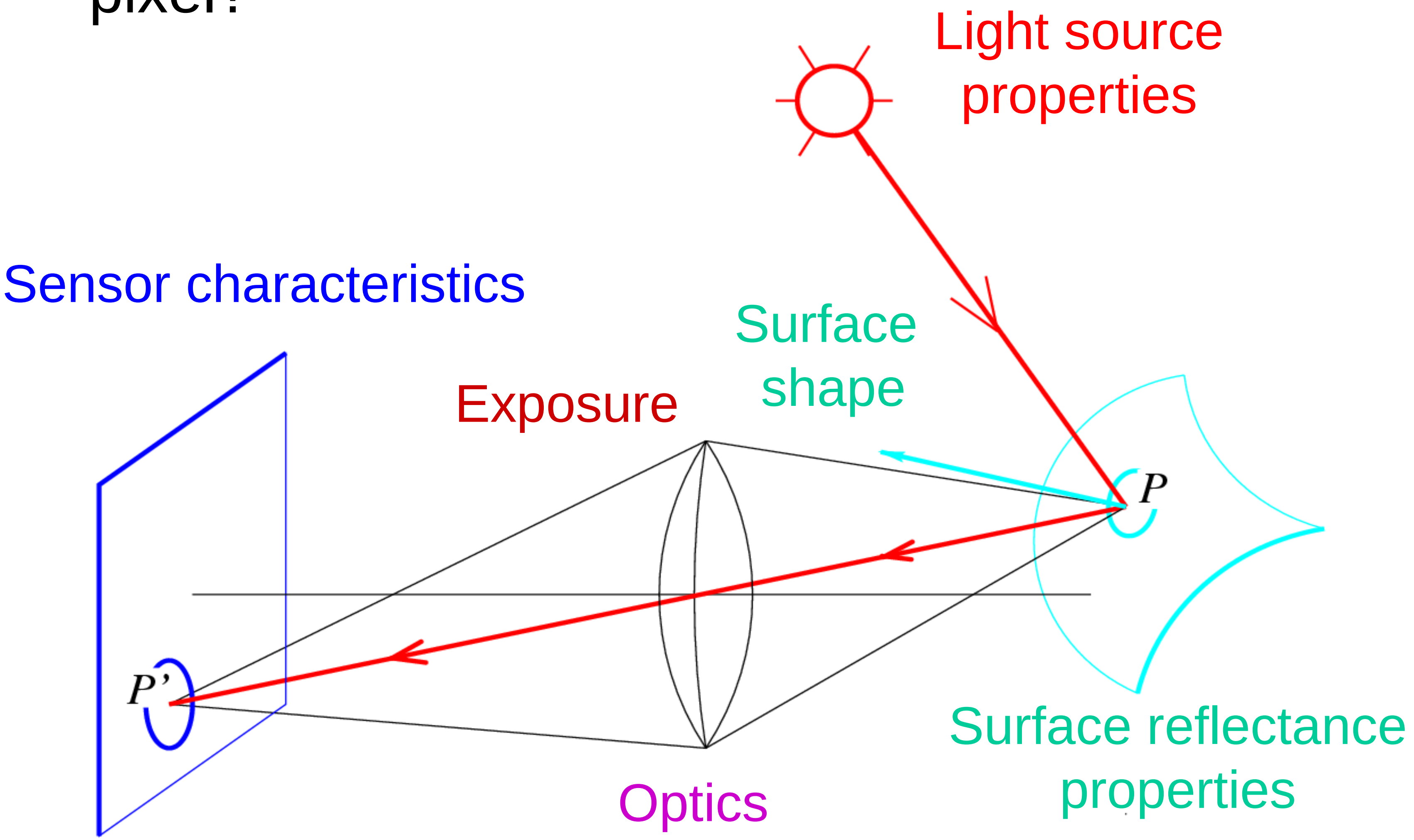
http://www.loc.gov/exhibits/empire/

Today

- Radiometry: measuring light
- Surface reflectance: BRDF
- Lambertian and specular surfaces
- Shape from shading
- Photometric stereo

Radiometry

What determines the brightness of an image pixel?



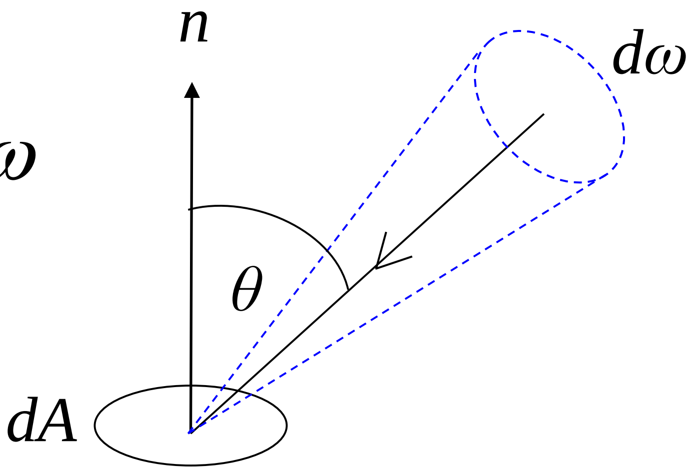
Slide by L. Fei-Fei

Radiometry

- Radiance (L): energy carried by a ray
  - Power per unit area perpendicular to the direction of travel, per unit solid angle
  - Units: Watts per square meter per steradian (W m^{-2} sr^{-1})

- Irradiance (E): energy arriving at a surface
  - Incident power in a given direction per unit area
  - Units: W m^{-2}
  - For a surface receiving radiance $L(x,\theta,\phi)$ coming in from dω the corresponding irradiance is

$$E(\theta,\phi) = L(\theta,\phi)\cos\theta \, d\omega$$



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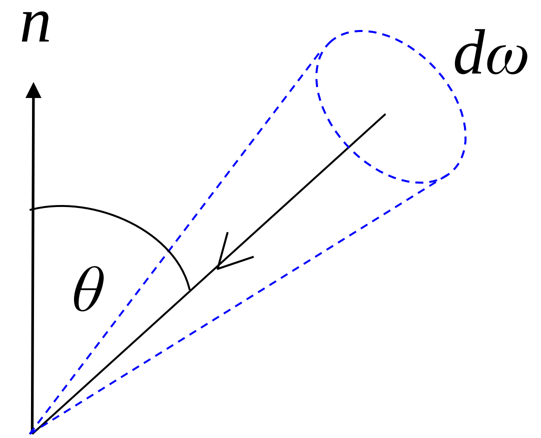

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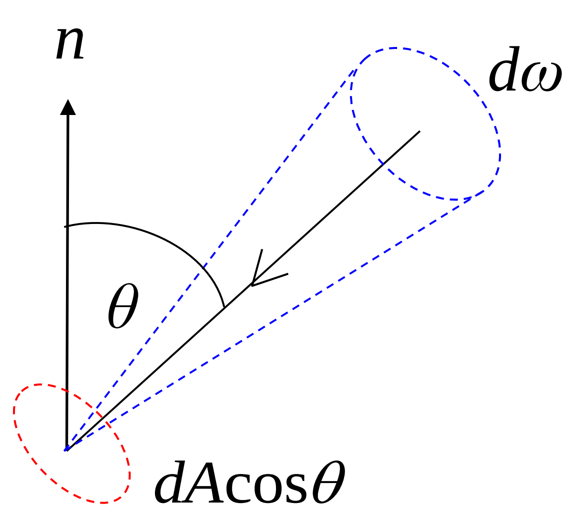
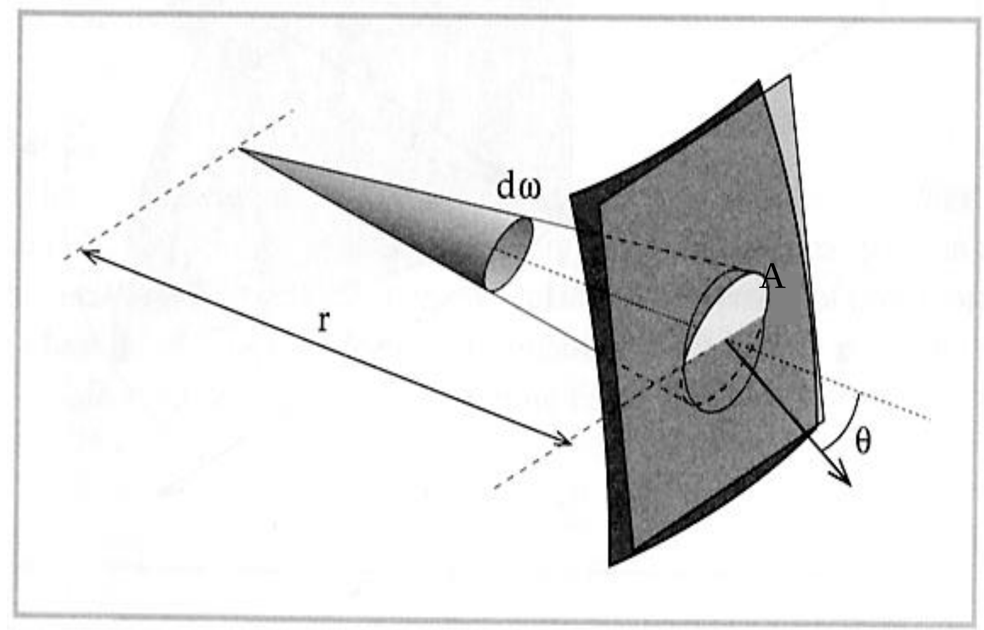


Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle $d\omega$ subtended by a patch of area *dA* is given by:

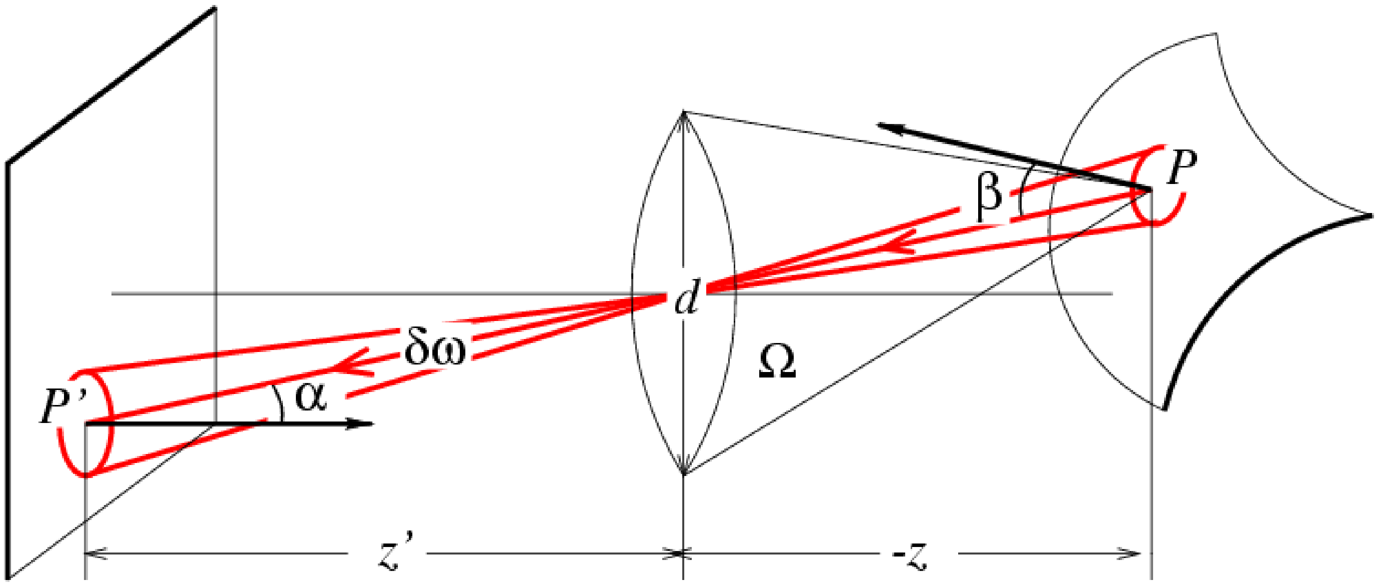
$$d\omega = \frac{dA\cos\theta}{r^2}$$



Radiometry of thin lenses

L: Radiance emitted from *P* toward *P*'

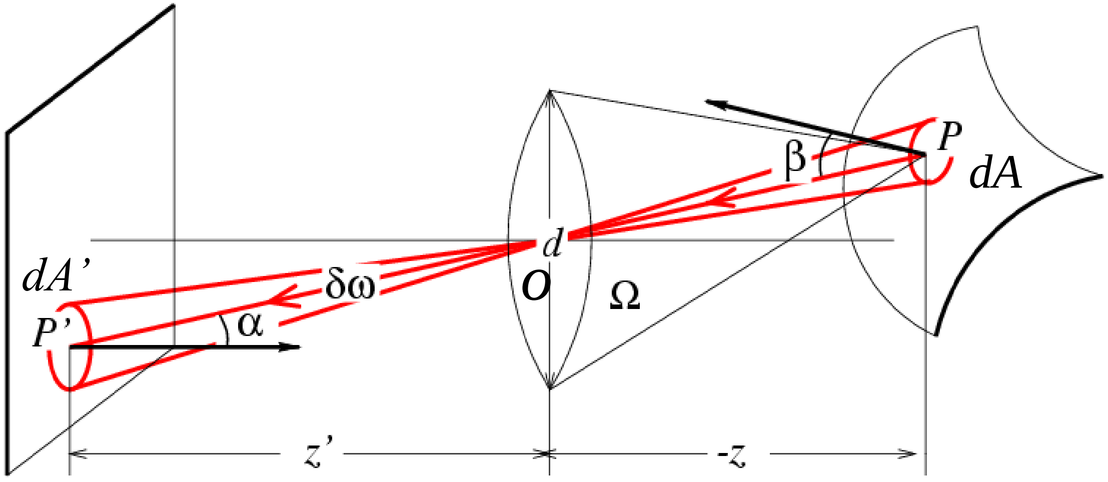
E: Irradiance falling on *P*' from the lens



What is the relationship between *E* and *L*?

Forsyth & Ponce, Sec. 4.2.3

Example: Radiometry of thin lenses

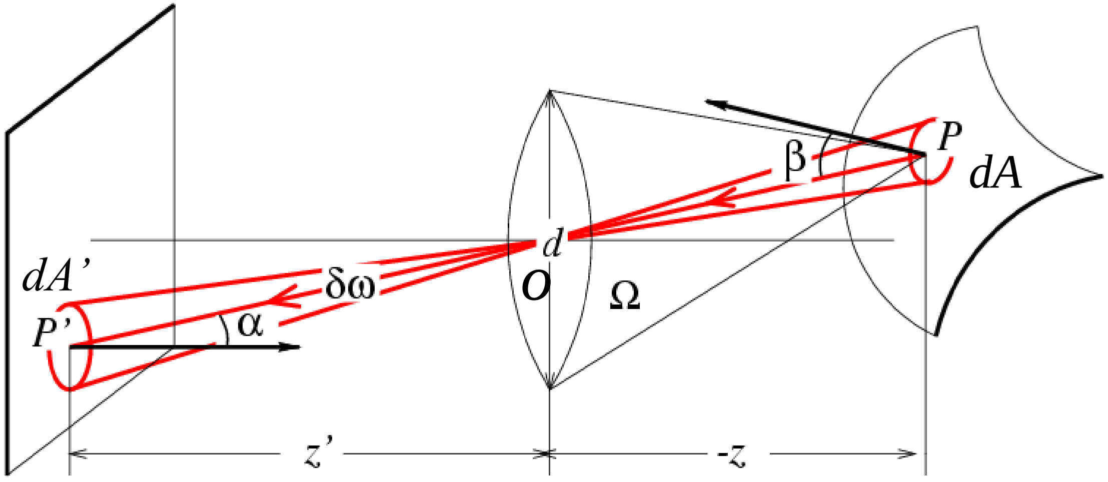


$$|OP| = \frac{z}{\cos\alpha}$$

$$|OP'| = \frac{z'}{\cos\alpha}$$

Area of the lens: $\dfrac{\pi d^2}{4}$

Example: Radiometry of thin lenses



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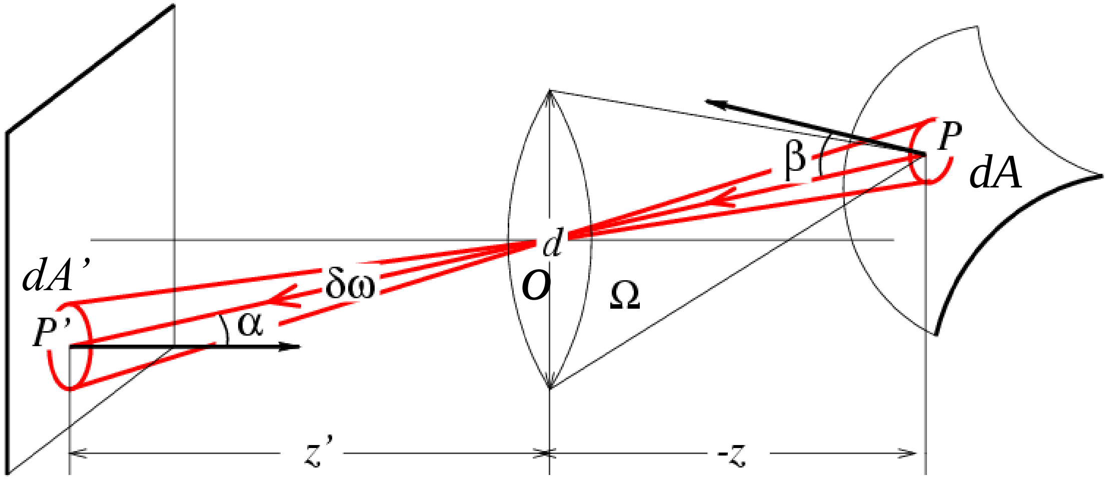

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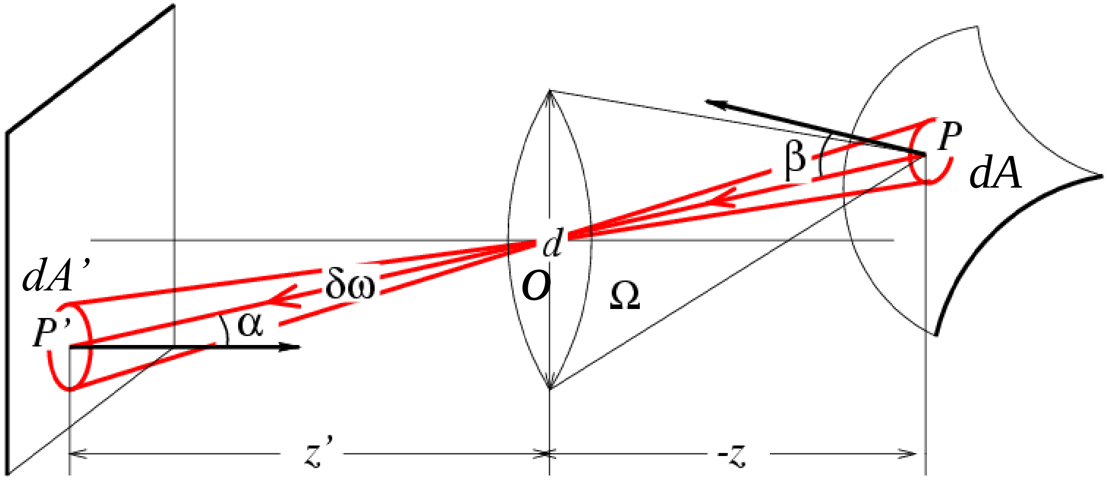

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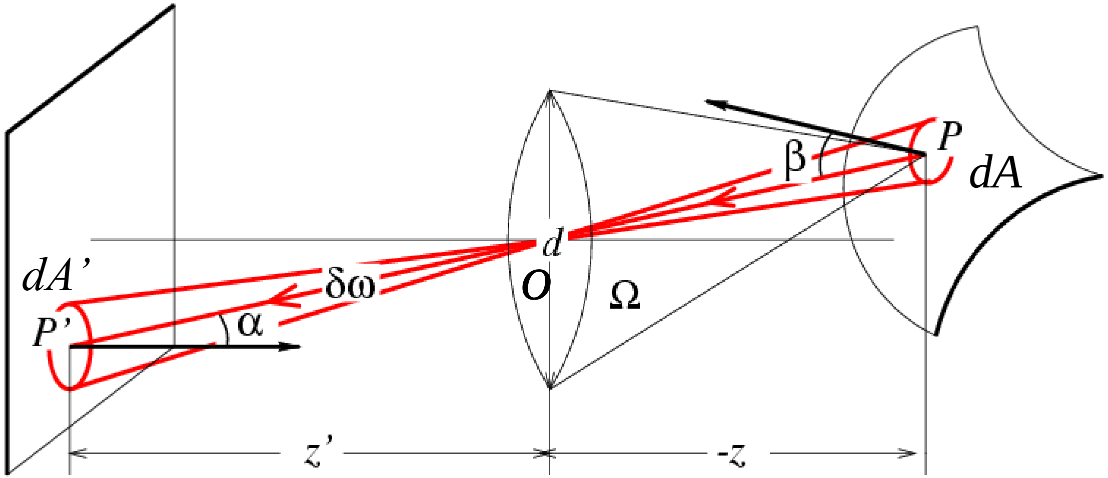

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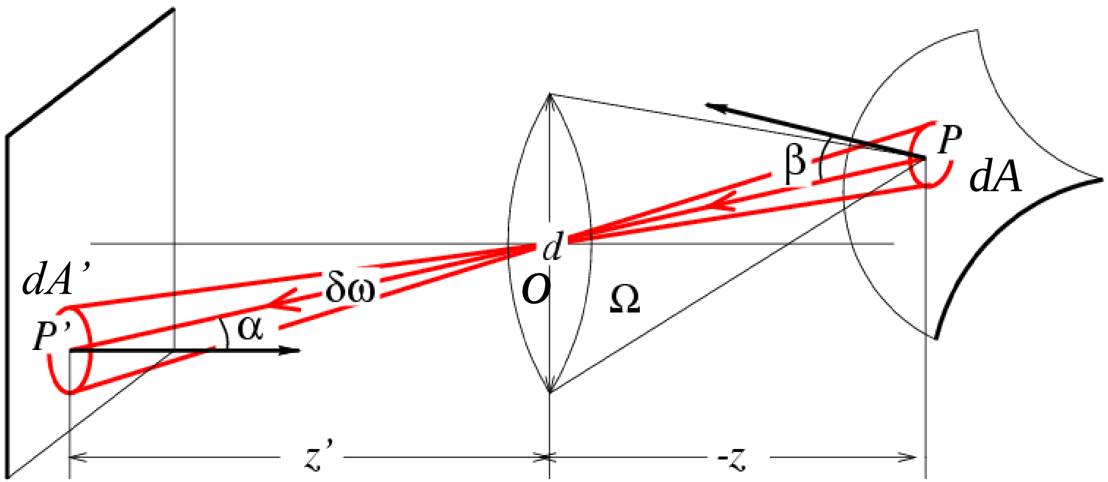

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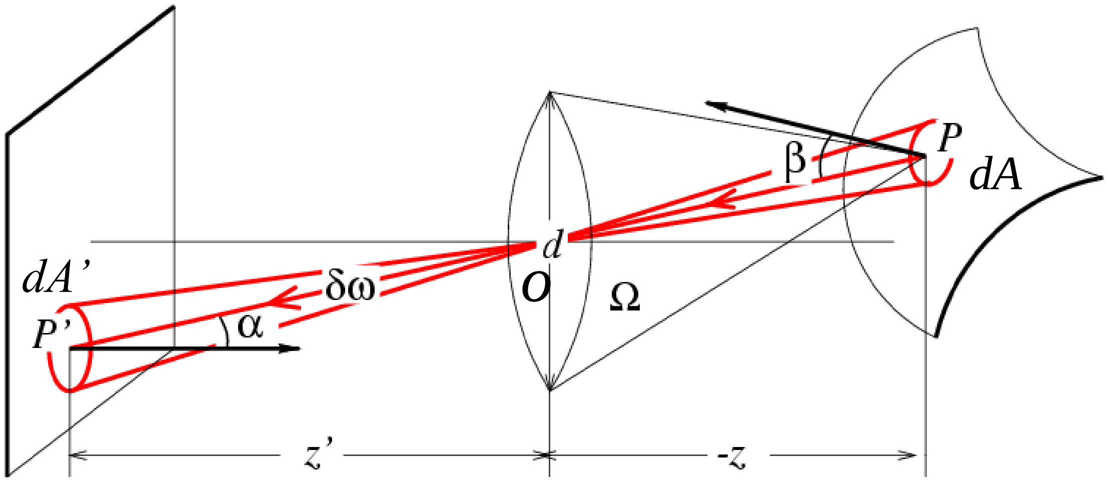

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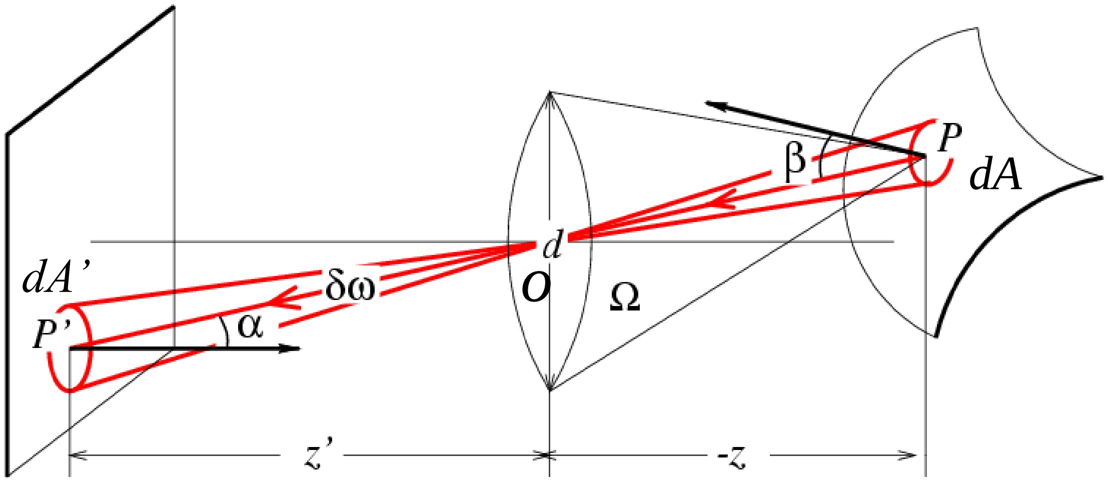

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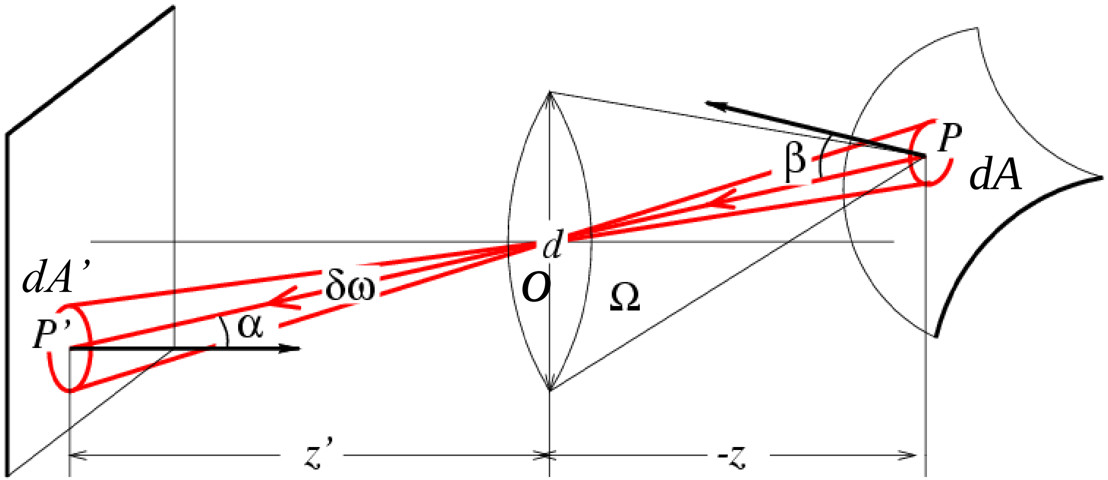

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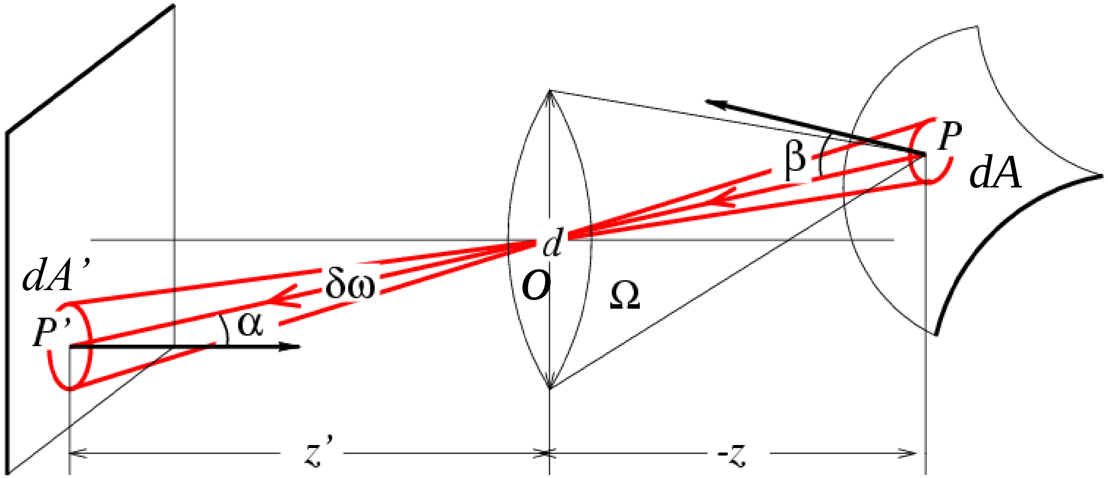

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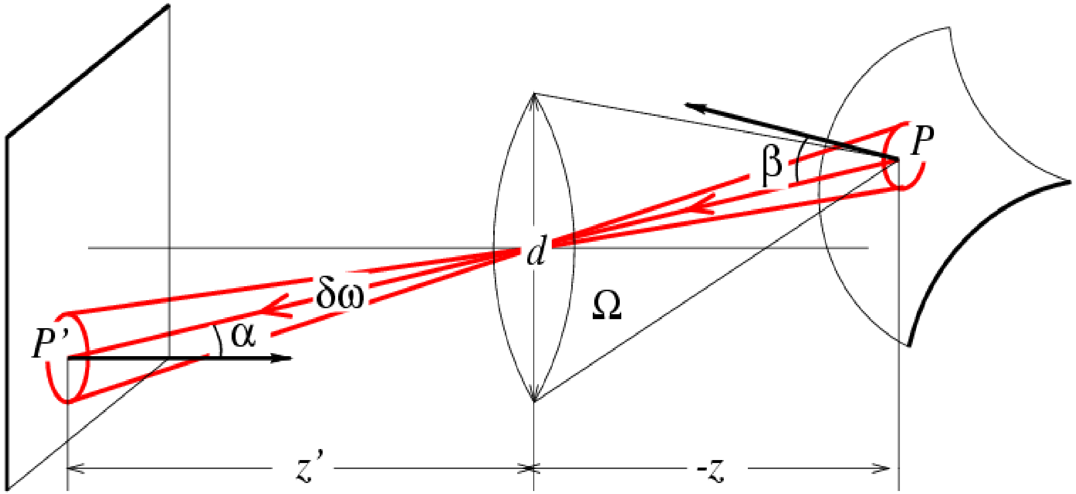
$$E = \left[\frac{\pi}{4}\left(\frac{d}{z'}\right)^2\cos^4\alpha\right]L$$

Radiometry of thin lenses



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

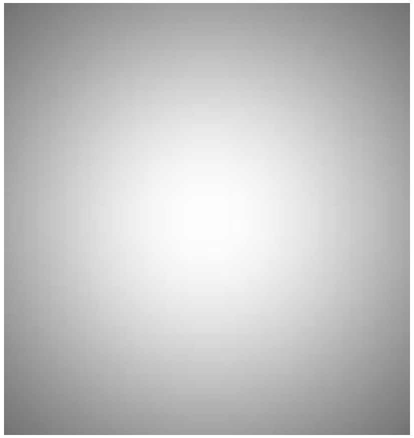
- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Forsyth & Ponce, Sec. 4.2.3

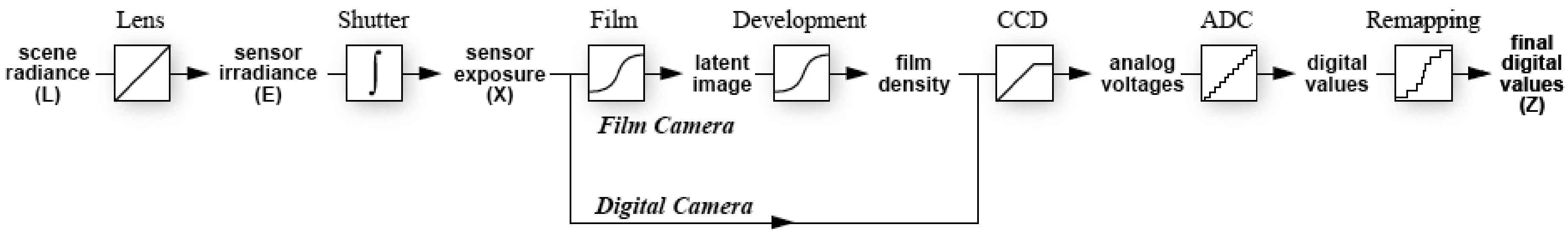
Radiometry of thin lenses

$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

- Application:
  - S. B. Kang and R. Weiss, *Can we calibrate a camera using an image of a flat, textureless Lambertian surface?* ECCV 2000.



The journey of the light ray



Source: S. Seitz, P. Debevec

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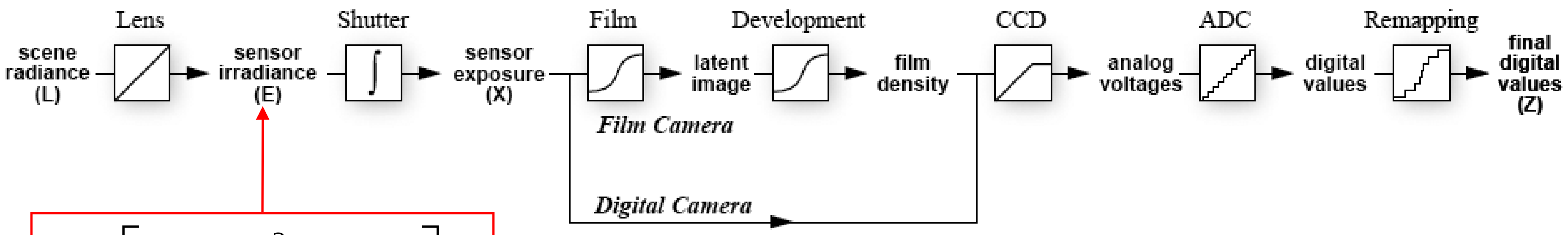

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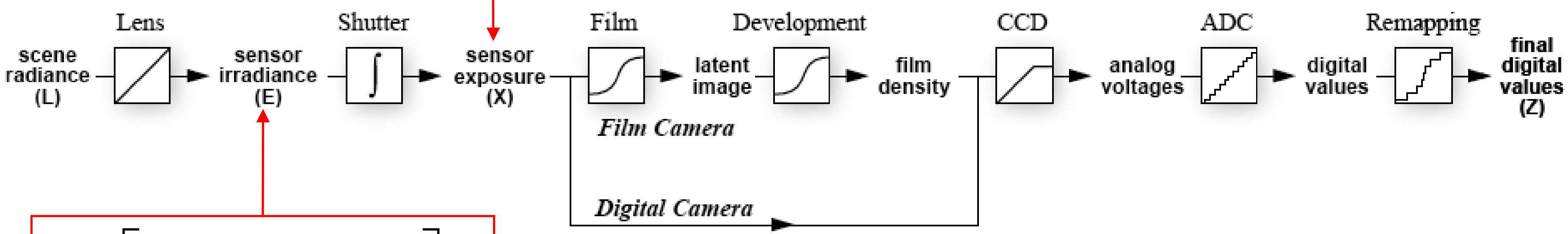
The journey of the light ray

$$X = E \cdot \Delta t$$



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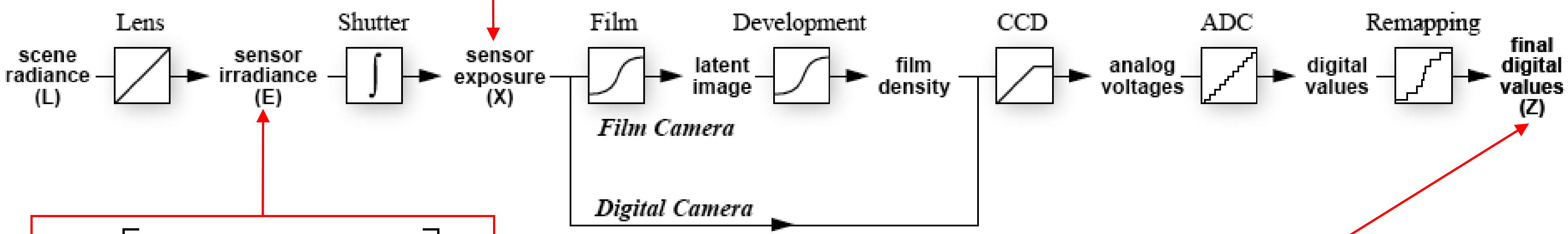


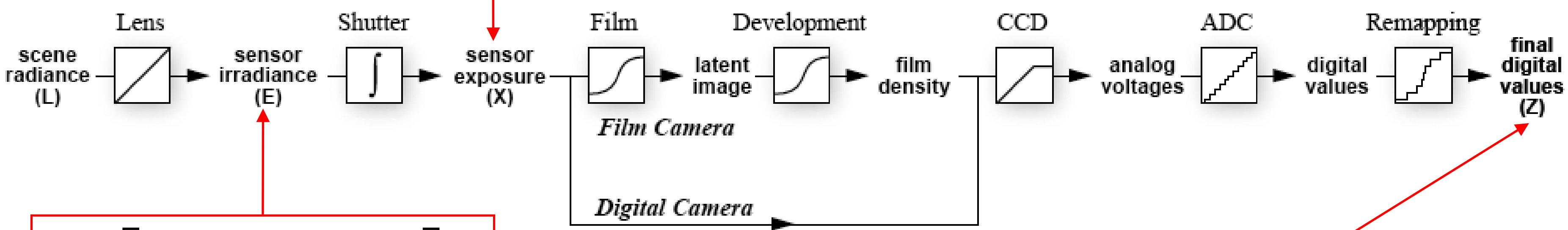
$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

$$Z = f\left(E \cdot \Delta t\right)$$

Source: S. Seitz, P. Debevec

The journey of the light ray

$$X = E \cdot \Delta t$$



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

$$Z = f\left(E \cdot \Delta t\right)$$

- Camera response function: the mapping f from irradiance to pixel values
  - Useful if we want to estimate material properties
  - Shape from shading requires irradiance
  - Enables us to create high dynamic range images

Source: S. Seitz, P. Debevec

Recovering the camera response function

- Method 1: Modeling
  - Carefully model every step in the pipeline
  - Measure aperture, model film, digitizer, etc.
  - This is *really* hard to get right

Slide by Steve Seitz

Recovering the camera response function

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- Method 2: Calibration
  - Take pictures of several objects with known irradiance
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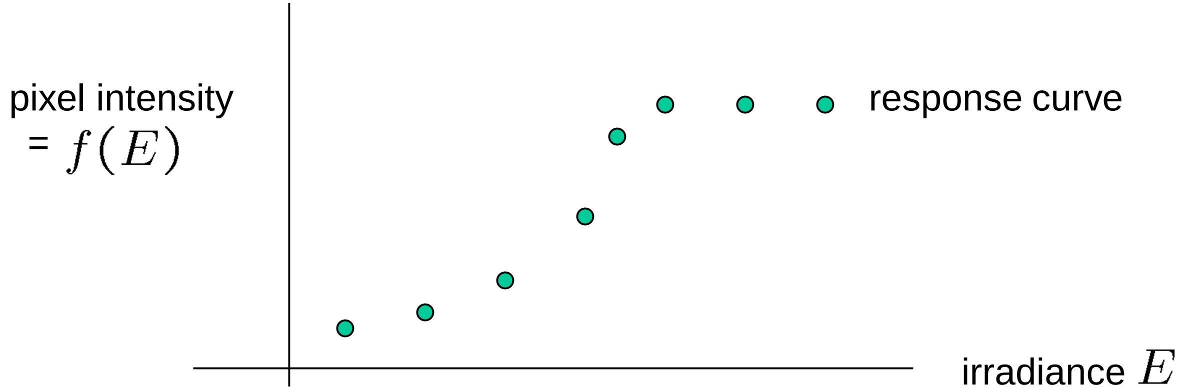
  - Fit a function



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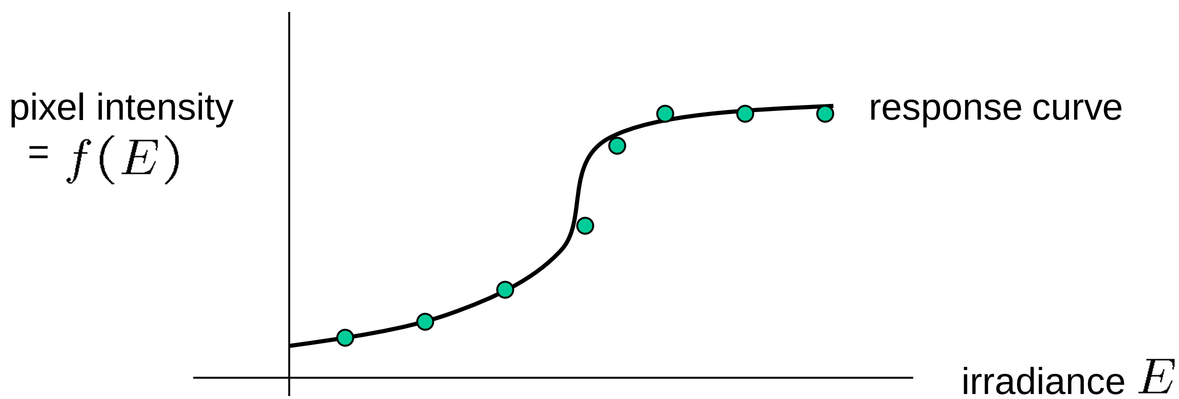
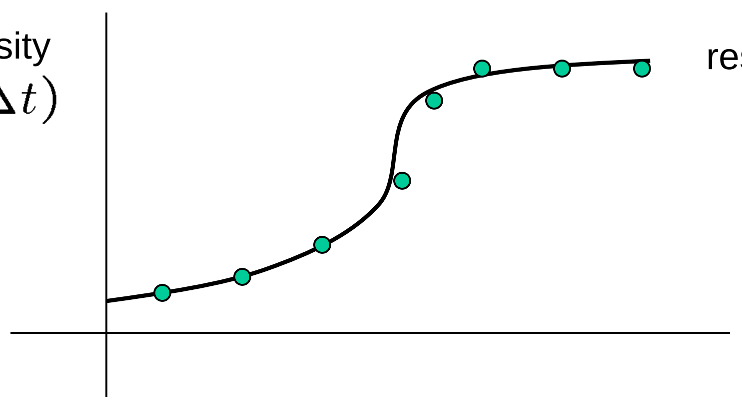


Slide by Steve Seitz

Recovering the camera response function

Method 3: Multiple exposures

- Consider taking images with shutter speeds 1/1000, 1/100, 1/10, 1
- The sensor exposures in consecutive images get scaled by a factor of 10
- This is the same as observing values of the response function for a range of irradiances: $f(E)$, $f(10E)$, $f(100E)$, etc.
- Can fit a function to these successive values



For more info

- P. E. Debevec and J. Malik. *Recovering High Dynamic Range Radiance Maps from Photographs*. In *SIGGRAPH 97*, August 1997

Slide by Steve Seitz

The interaction of light and matter

What happens when a light ray hits a point on an object?

- Some of the light gets absorbed
  - converted to other forms of energy (e.g., heat)
- Some gets transmitted through the object
  - possibly bent, through “refraction”
- Some gets reflected
  - possibly in multiple directions at once
- Really complicated things can happen
  - fluorescence

Slide by Steve Seitz

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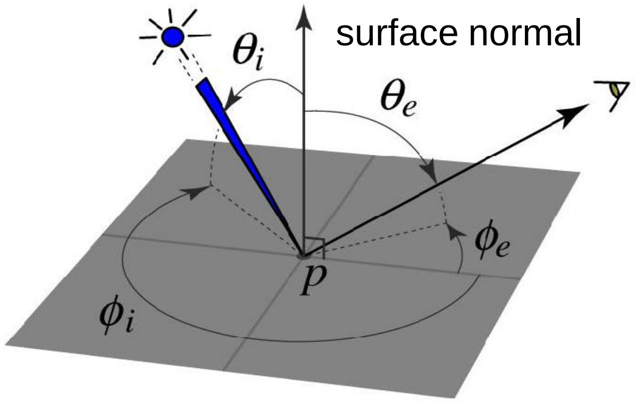
Let’s consider the case of reflection in detail

- In the most general case, a single incoming ray could be reflected in all directions. How can we describe the amount of light reflected in each direction?

Slide by Steve Seitz

Bidirectional reflectance distribution function (BRDF)

- Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the outgoing direction to irradiance in the incident direction

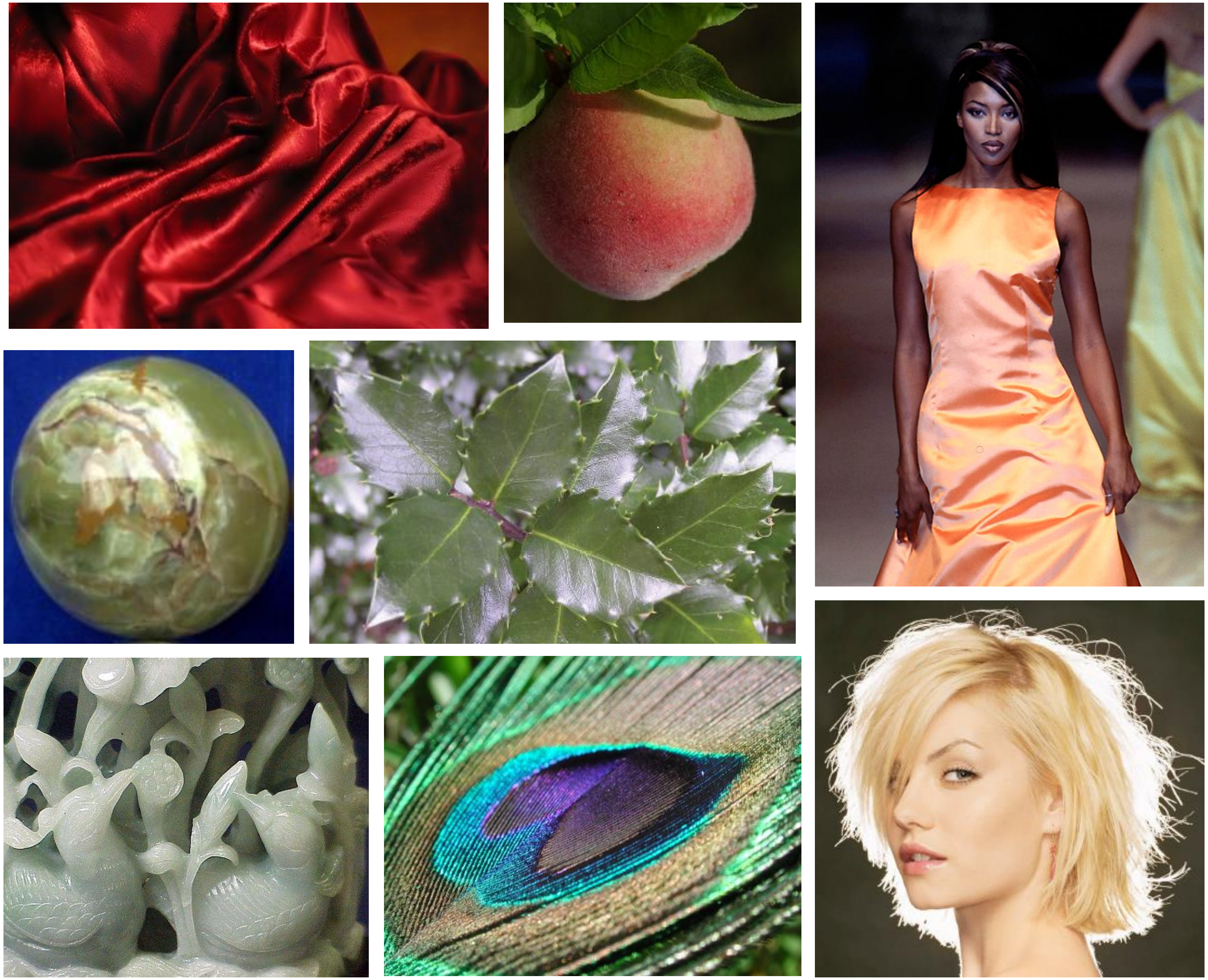


$$\rho(\theta_i,\phi_i,\theta_e,\phi_e) = \frac{L_e(\theta_e,\phi_e)}{E_i(\theta_i,\phi_i)} = \frac{L_e(\theta_e,\phi_e)}{L_i(\theta_i,\phi_i)\cos\theta_i d\omega}$$

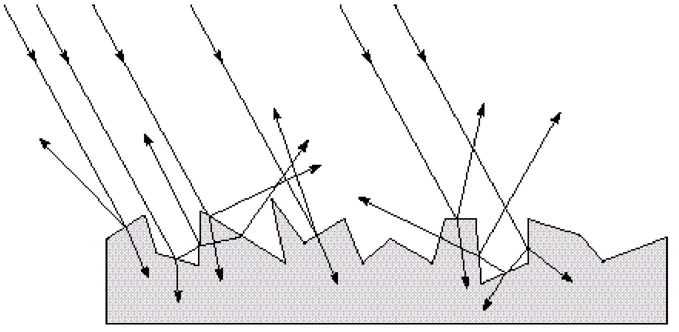
- Radiance leaving a surface in a particular direction: add contributions from every incoming direction

$$\int_{\Omega} \rho(\theta_i,\phi_i,\theta_e,\phi_e,)L_i(\theta_i,\phi_i)\cos\theta_i d\omega_i$$

BRDF’s can be incredibly complicated…



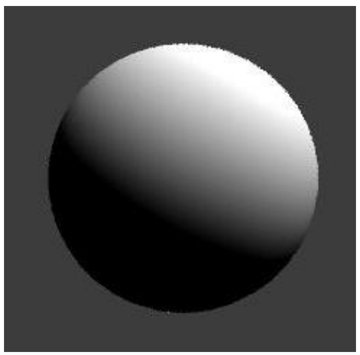
Diffuse reflection

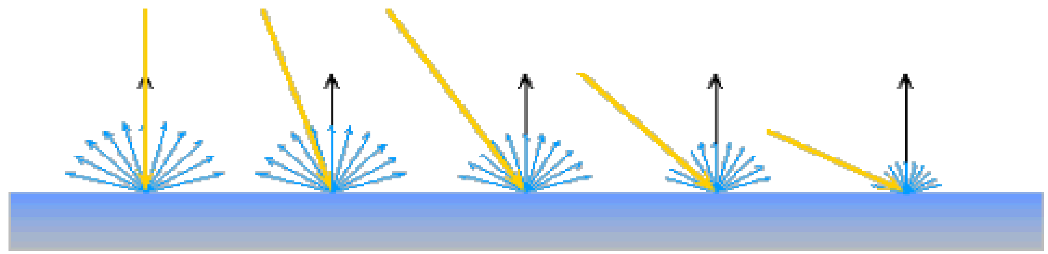


- Dull, matte surfaces like chalk or latex paint
- Microfacets scatter incoming light randomly
- Light is reflected equally in all directions: **BRDF is constant**
- *Albedo*: fraction of incident irradiance reflected by the surface
- *Radiosity:* total power leaving the surface per unit area (regardless of direction)

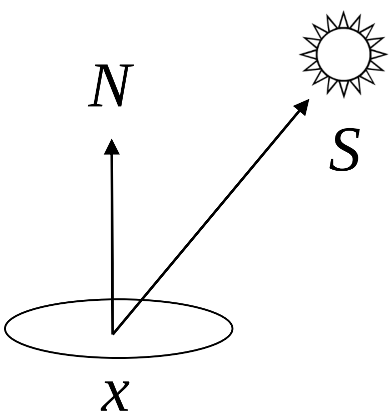
Diffuse reflection: Lambert's law

- Viewed brightness does not depend on viewing direction, but it *does* depend on direction of illumination





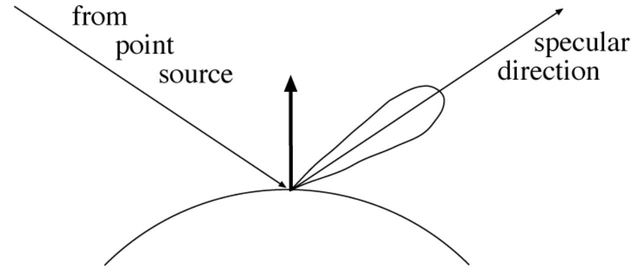
$$B(x) = \rho_d(x)\left(N(x)\cdot S_d(x)\right)$$

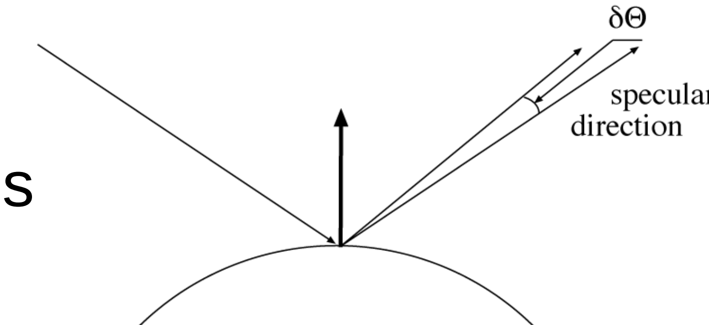


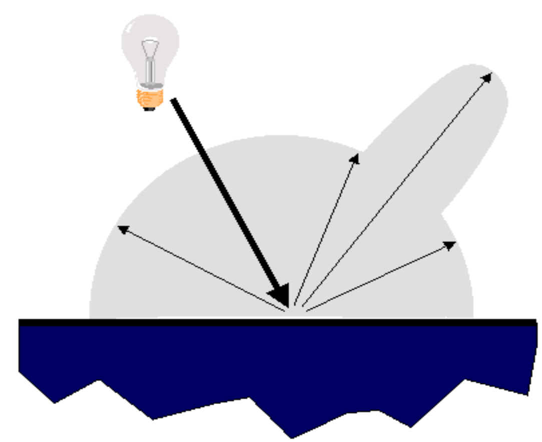
B: radiosity
ρ: albedo
N: unit normal
S: source vector (magnitude proportional to intensity of the source)

Specular reflection

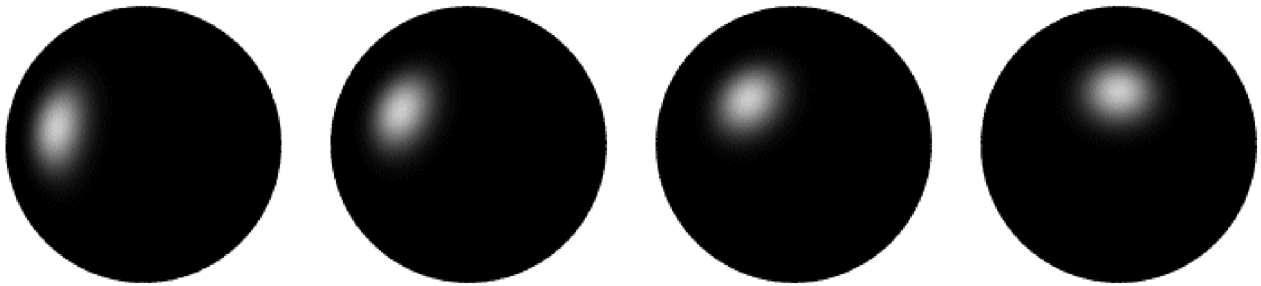
- Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
- Some fraction is absorbed, some reflected
- On real surfaces, energy usually goes into a lobe of directions
- Phong model: reflected energy falls of with $\cos^n(\delta\theta)$
- Lambertian + specular model: sum of diffuse and specular term



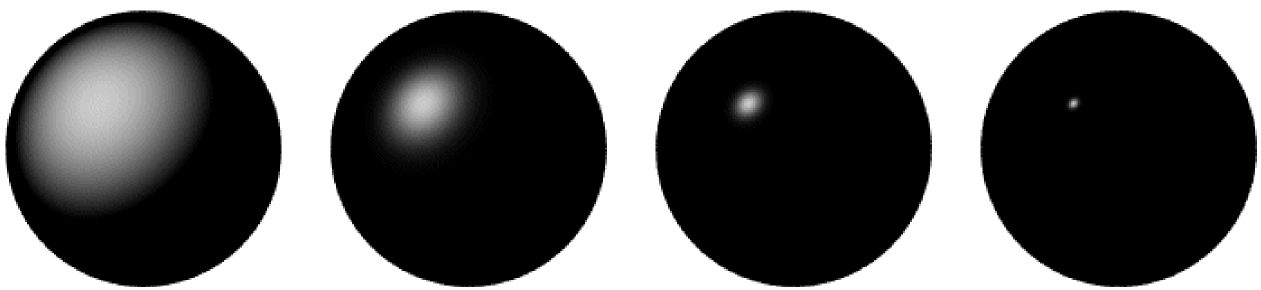




Specular reflection



Moving the light source



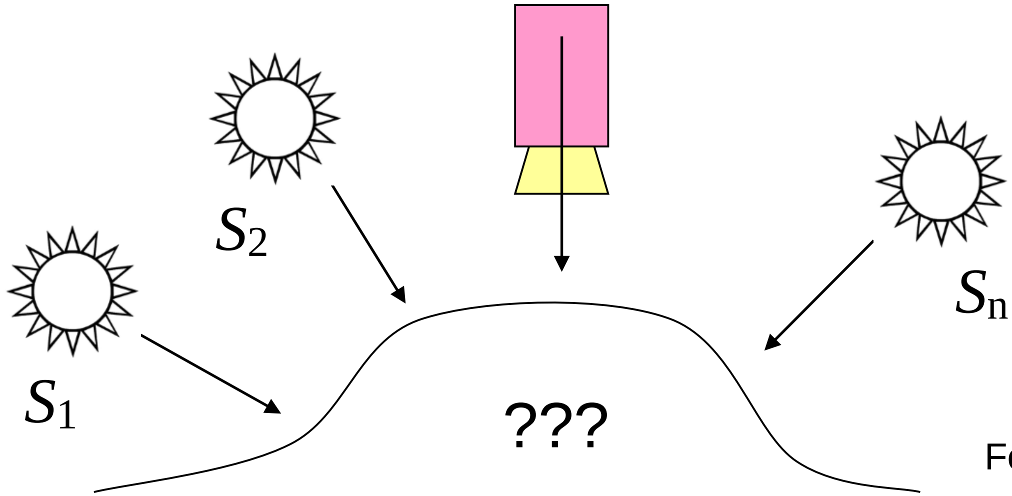
Changing the exponent

Photometric stereo

Assume:

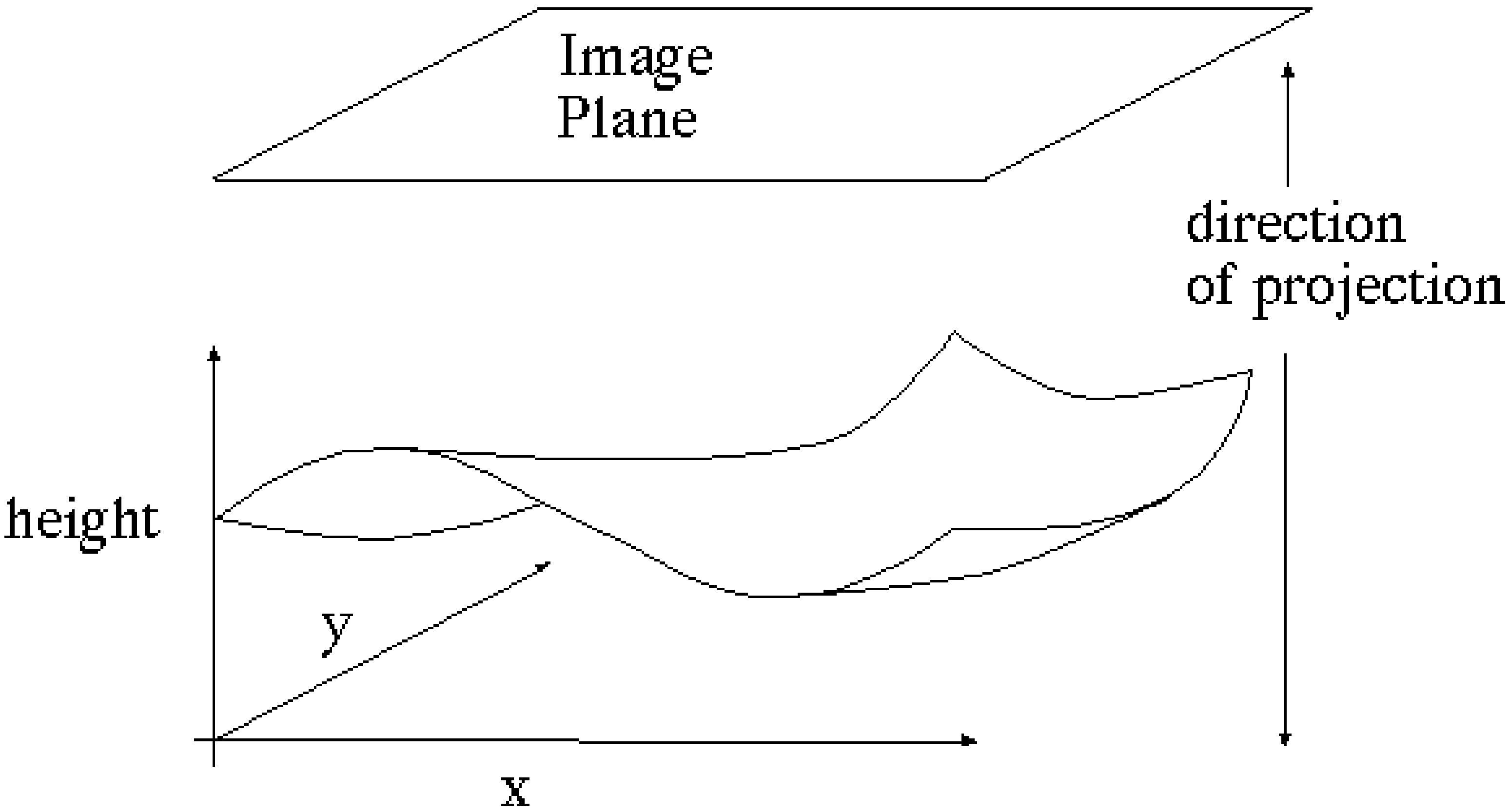
- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo



Forsyth & Ponce, Sec. 5.4

Surface model: Monge patch



Forsyth & Ponce, Sec. 5.4

Image model

- Known: source vectors S_j and pixel values $I_j(x,y)$
- We also assume that the response function of the camera is a linear scaling by a factor of k
- Combine the unknown normal $N(x,y)$ and albedo $\rho(x,y)$ into one vector g, and the scaling constant and source vectors into another vector V_j:

$$
\begin{aligned}
I_j(x,y) &= k\,B(x,y) \\
&= k\,\rho(x,y)\big(N(x,y)\cdot S_j\big) \\
&= \big(\rho(x,y)N(x,y)\big)\cdot(kS_j) \\
&= g(x,y)\cdot V_j
\end{aligned}
$$

Forsyth & Ponce, Sec. 5.4

Least squares problem

- For each pixel, we obtain a linear system:

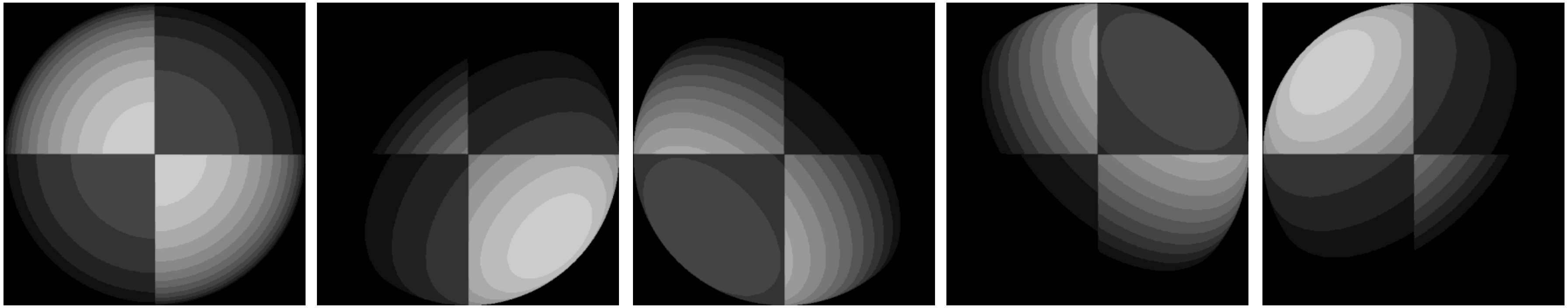
$$\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix} = \begin{bmatrix} V_1^{\,T} \\ V_2^{\,T} \\ \vdots \\ V_n^{\,T} \end{bmatrix} g(x,y)$$

$(n \times 1)$	$(n \times 3)$	(3×1)
known	known	unknown

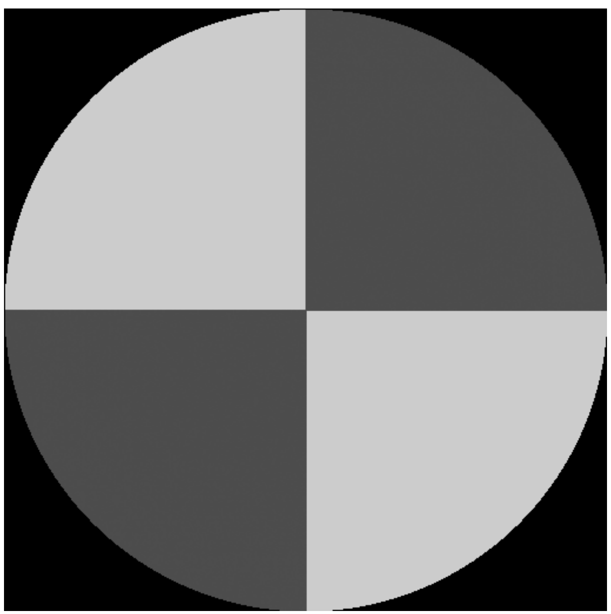
- Obtain least-squares solution for $g(x,y)$
- Since $N(x,y)$ is the unit normal, $\rho(x,y)$ is given by the magnitude of $g(x,y)$ (and it should be less than 1)
- Finally, $N(x,y) = g(x,y) / \rho(x,y)$

Forsyth & Ponce, Sec. 5.4

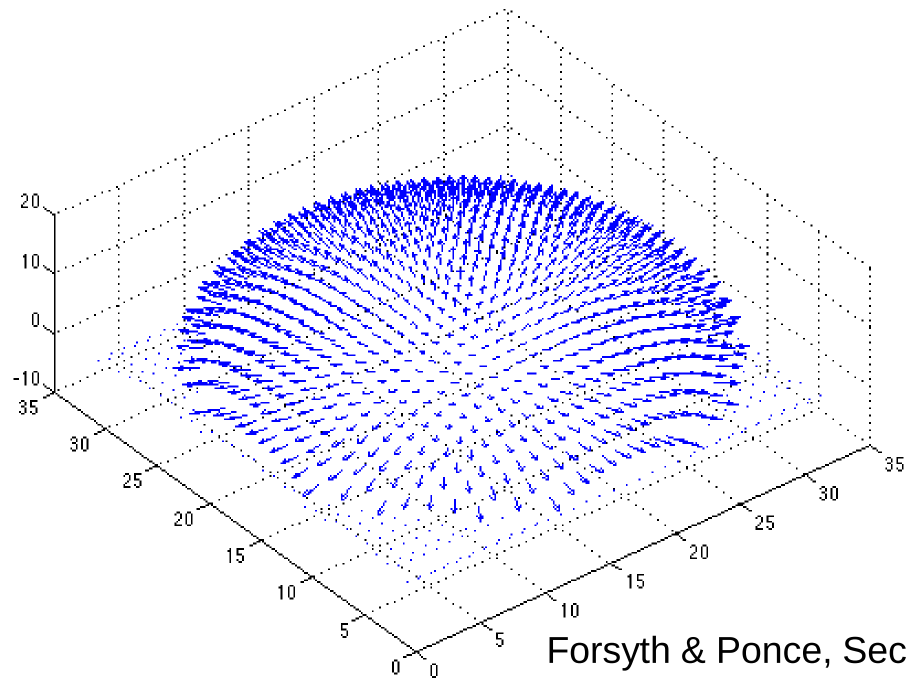
Example



Recovered albedo



Recovered normal field



Forsyth & Ponce, Sec. 5.4

Recovering a surface from normals - 1

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$N(x, y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \right) \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

If we write the known vector *g* as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = \left(g_1(x, y) / g_3(x, y) \right)$$

$$f_y(x, y) = \left(g_2(x, y) / g_3(x, y) \right)$$

Forsyth & Ponce, Sec. 5.4

Recovering a surface from normals - 2

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial\left(g_1(x,y)/g_3(x,y)\right)}{\partial y} = \frac{\partial\left(g_2(x,y)/g_3(x,y)\right)}{\partial x}$$

(in practice, they should at least be similar)

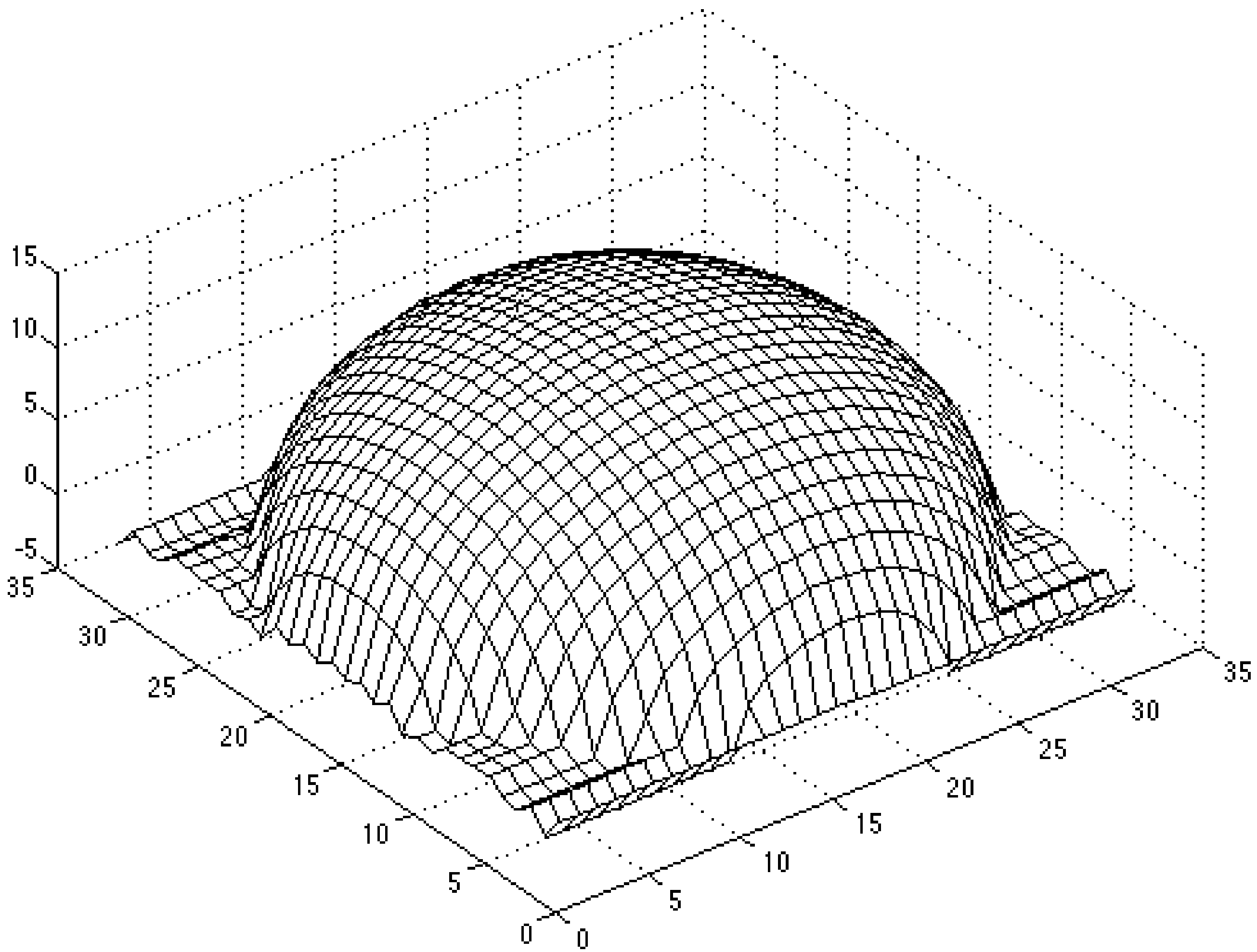
We can now recover the surface height at any point by integration along some path, e.g.

$$f(x,y) = \int_0^x f_x(s,y)\,ds + \int_0^y f_y(x,t)\,dt + c$$

(for robustness, can take integrals over many different paths and average the results)

Forsyth & Ponce, Sec. 5.4

Surface recovered by integration



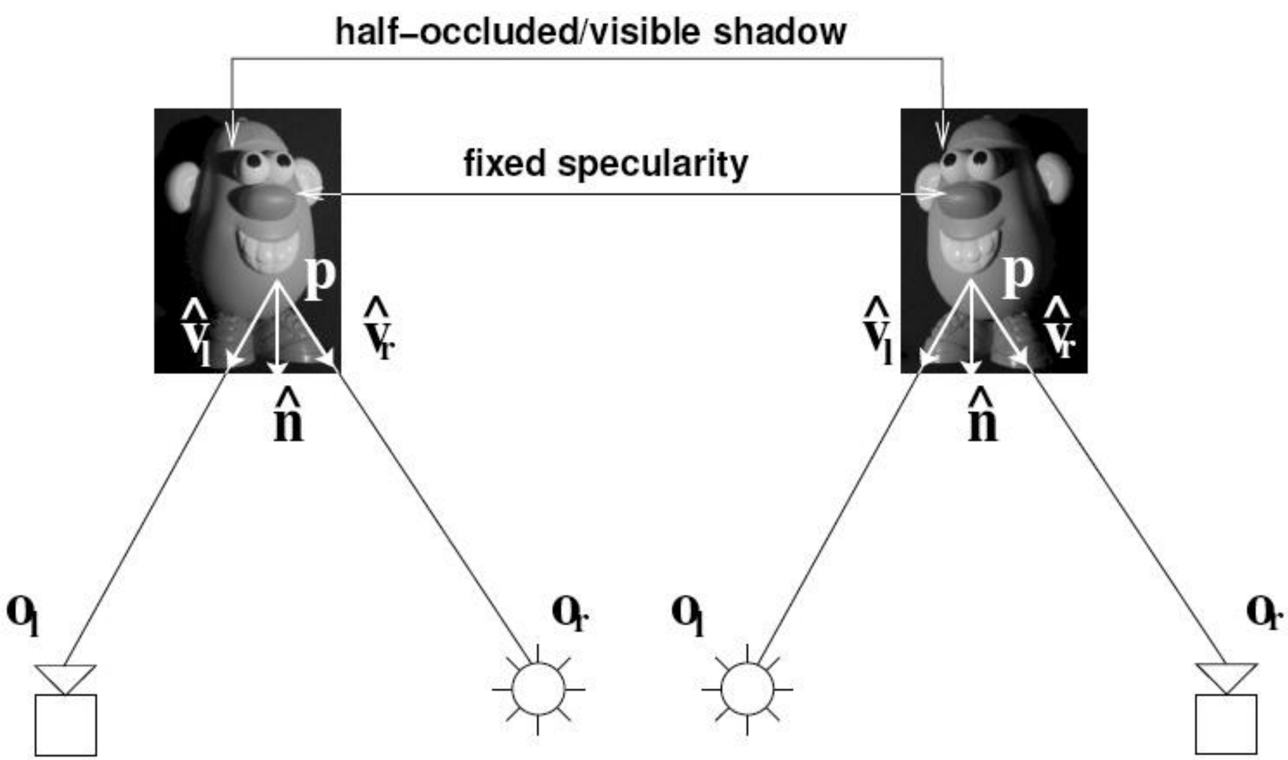
Forsyth & Ponce, Sec. 5.4

Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

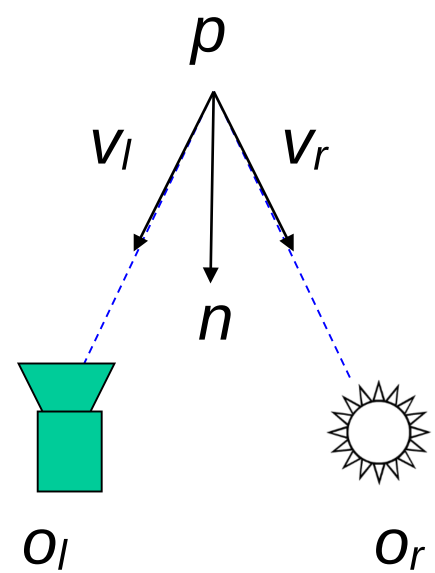
Reconstructing surfaces with arbitrary BRDF's

- T. Zickler, P. Belhumeur, and D. Kriegman, "Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction," ECCV 2002.
- Key idea: switch the camera and the light source



Helmholtz stereopsis

- Let's put the light at o_r and the camera at o_l
- Recall that the BRDF $\rho(v_l,v_r)$ is the ratio of outgoing radiance in direction v_l to incident irradiance in direction v_r

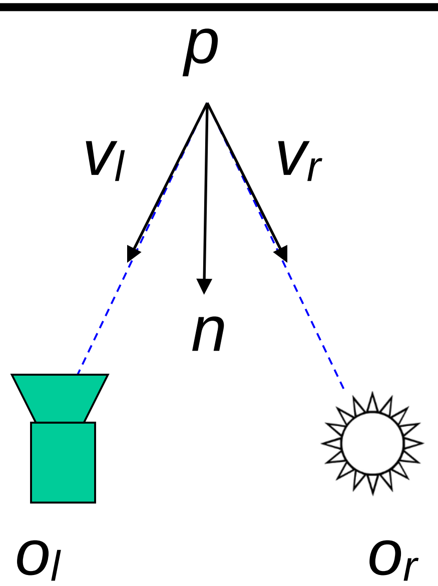


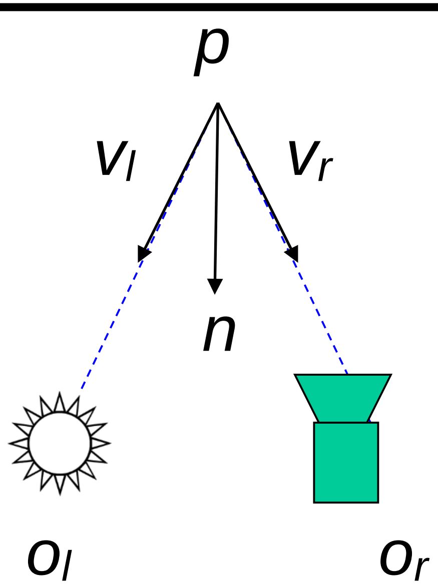
$$\rho(v_r, v_l) = \frac{\eta I_l}{\dfrac{n \cdot v_r}{|o_r - p|^2}}$$

Outgoing radiance: proportional to observed image irradiance

Incident irradiance: radiance received from the light multiplied by the foreshortening (cosine) term and by the solid angle ($1/d^2$) term

Helmholtz stereopsis

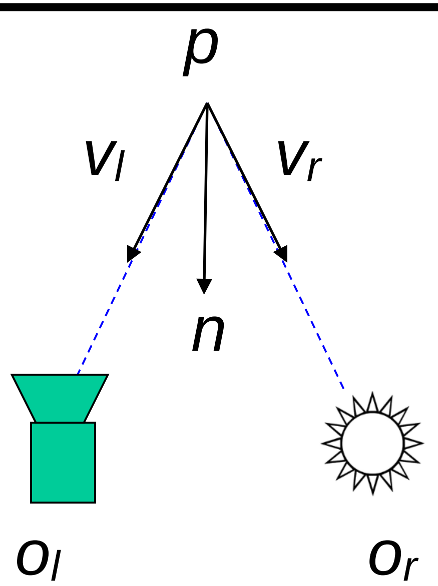


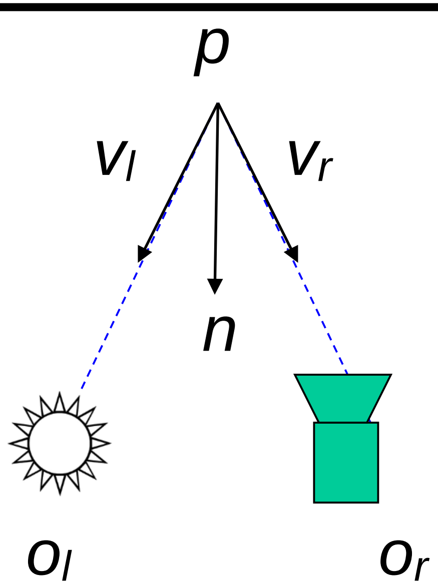


$$\rho(v_r, v_l) = \frac{\eta I_l}{\frac{n \cdot v_r}{|o_r - p|^2}}$$

$$\rho(v_l, v_r) = \frac{\eta I_r}{\frac{n \cdot v_l}{|o_l - p|^2}}$$

Helmholtz stereopsis





$$\rho(v_r, v_l) = \frac{\eta I_l}{\frac{n \cdot v_r}{|o_r - p|^2}}$$

$$\rho(v_l, v_r) = \frac{\eta I_r}{\frac{n \cdot v_l}{|o_l - p|^2}}$$

- Helmholtz reciprocity: $\rho(v_l, v_r) = \rho(v_r, v_l)$

Helmholtz stereopsis

$$\frac{\eta I_l}{\frac{n \cdot v_r}{|o_r - p|^2}} = \frac{\eta I_r}{\frac{n \cdot v_l}{|o_l - p|^2}}$$

$$I_l \frac{n \cdot v_l}{|o_l - p|^2} = I_r \frac{n \cdot v_r}{|o_r - p|^2}$$

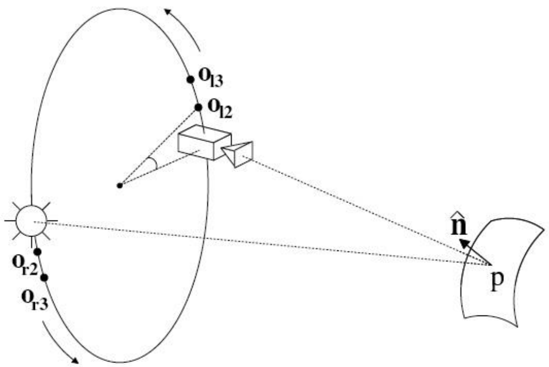
$$\left(\frac{I_l v_l}{|o_l - p|^2} - \frac{I_r v_r}{|o_r - p|^2} \right) \cdot n = 0$$

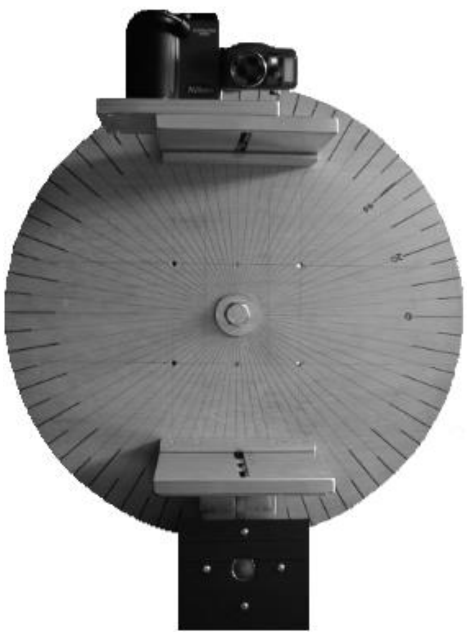
$$w(d) \cdot n = 0$$

Helmholtz stereopsis

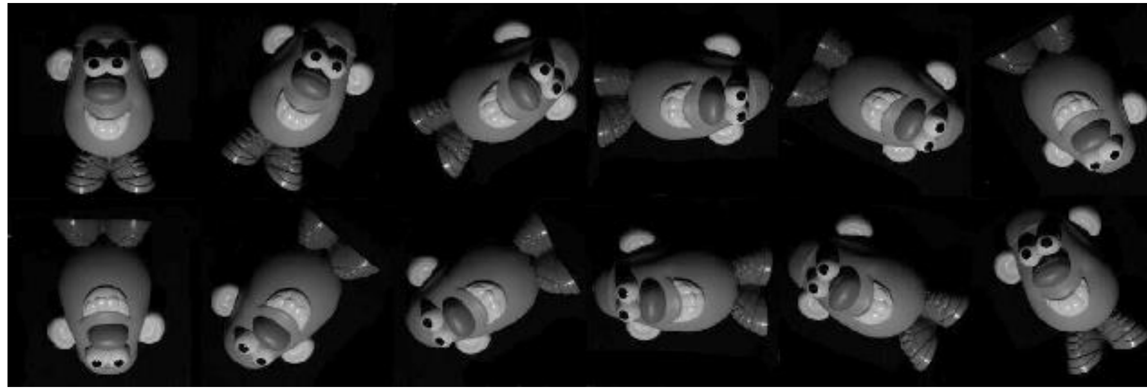
- The expression $w(d) \cdot n = 0$ provides a constraint both on the depth of the point and its normal
- We get M constraints for M light/camera pairs
- These constraints can be used for surface reconstruction: for example, we can search a range of depth values to determine which one best satisfies the constraints…

Example results



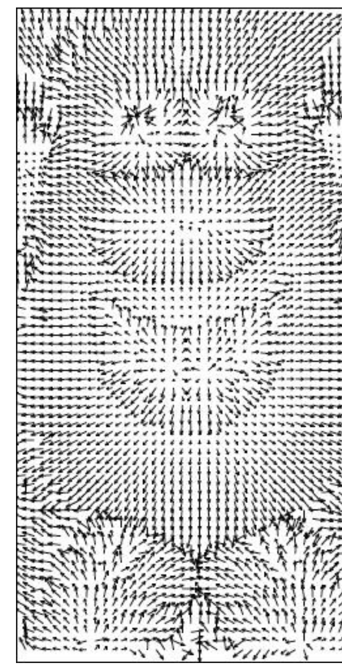


custom stereo rig



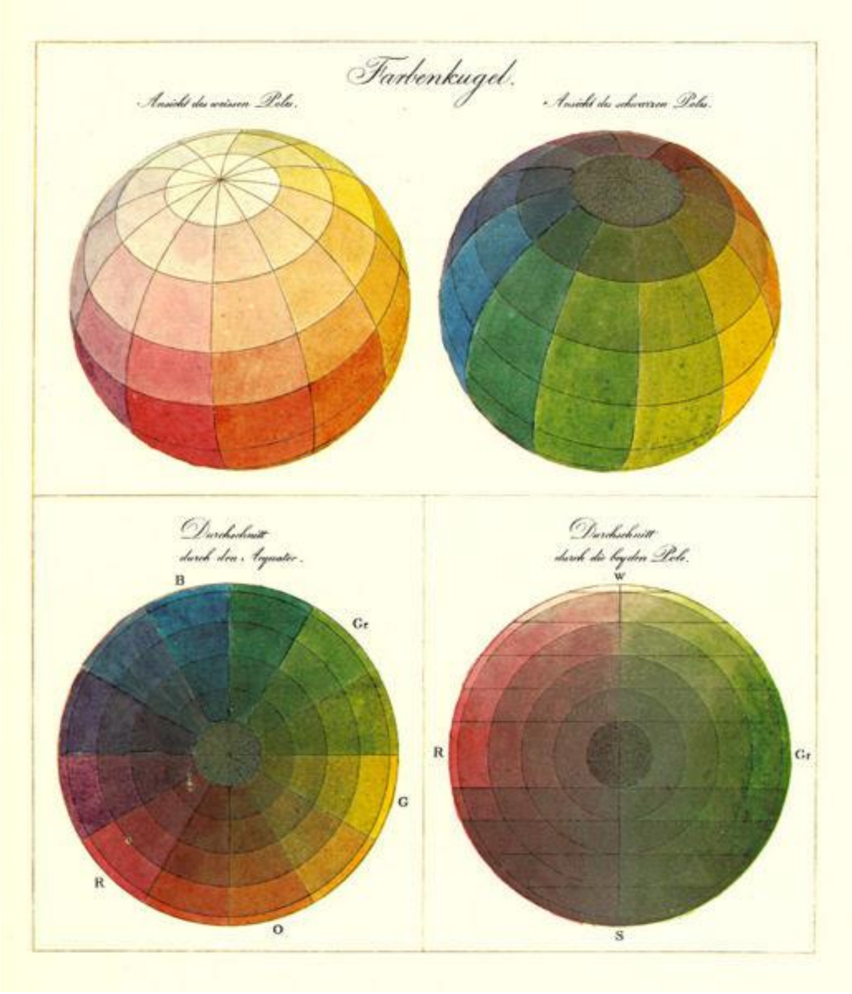
reciprocal stereo pairs

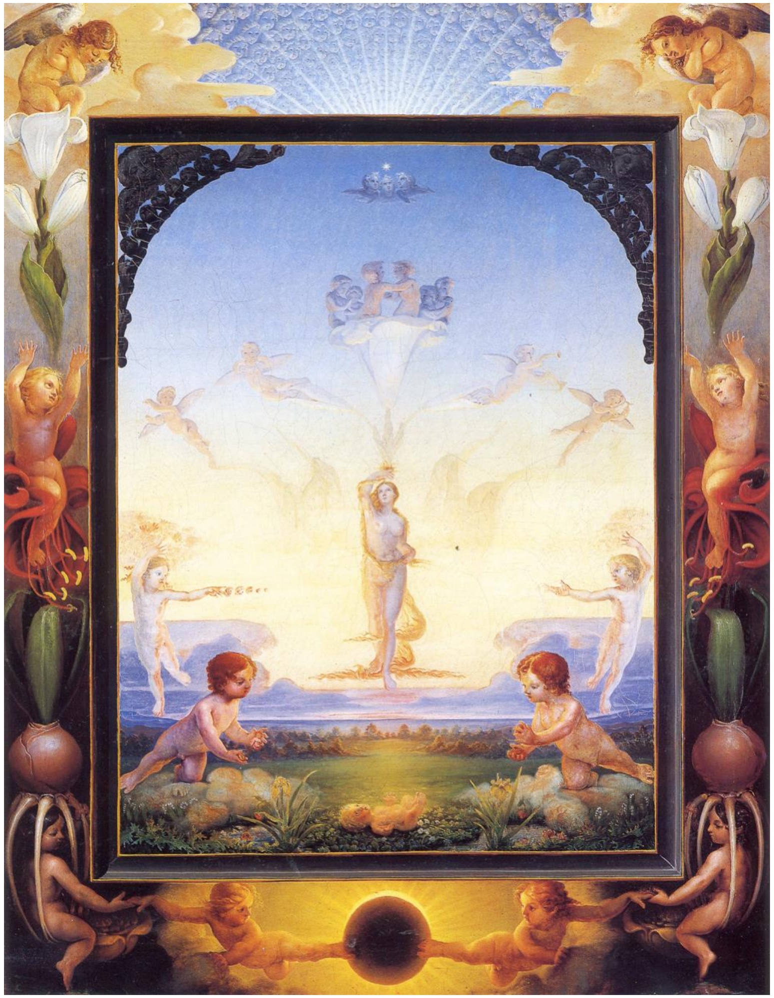
original image



recovered depth map and normal field

Next time: Color





Phillip Otto Runge (1777-1810)